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Information disclosure to Cournot duopolists[☆]

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HIGHLIGHTS

- A planner looks for the information structure that maximizes duopolists' surplus.
- The optimal policy is to fully inform one of them and say nothing to the other.
- The result extends in the oligopoly case but depends on specific assumptions.

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ABSTRACT

We show that in a standard symmetric Cournot duopoly with unknown demand, the optimal information disclosure policy of an informed benevolent planner is to fully inform one of the duopolists and disclose no information to the other one. We discuss possible extensions of the result.

1. Introduction

The outcome of agents' decisions often depends on a state of nature. The agents' possible private information on the state is a crucial parameter of their strategic interaction. Most studies in the literature assume specific, *exogenously given* information structures, in which, e.g., one agent is more informed than the others. This note is concerned with the *endogenous* determination of information structures, that is, with the design of an optimal information structure by an informed planner. In particular, we do not impose any *a priori* restrictions on the information structures.

We focus on a simple, yet familiar, environment: a standard Cournot duopoly with substitute goods, affine inverse demand and constant marginal costs. At the beginning of the game, the Cournot duopolists are uncertain about demand, which is only known to a benevolent planner who maximizes the sum of the duopolists' payoffs. We assume that the planner fully commits to an information transmission rule before observing the state of nature and that he cannot lie.[1] However, he can manipulate the accuracy of the transmitted information by increasing the number of states that he declares possible. More importantly, we allow the planner to send a private message to each of the duopolists, who thus end up playing a Bayesian game in which private information is endogenously generated by the planner's messages. Except for the possibly different messages from the planner, the duopolists are completely identical.

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1 As in Milgrom (1981), the planner can only manipulate the precision of his message.

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Our main finding (Proposition 1) is that, if demand is concentrated enough, it is optimal for the planner to treat the two duopolists *asymmetrically*: fully reveal the state of nature to one, while keeping the other in the dark. Thus, any requirement to transmit only public information (perhaps due to equity concerns) may come at the expense of ex-ante welfare.[2] We then show that Proposition 1 extends to the oligopoly case, but may fail if demand is dispersed, cost functions are quadratic, goods are not substitutes or if the planner's objective is not the producers' surplus.

Our objective in presenting these results is to encourage readers to extend our analysis and study the case without commitment.

Due to the lack of space, we limit references to a minimum. The closest paper to ours is Eliaz and Serrano (2014), which studies a similar question when the interacting agents are involved in a generalized multi-action prisoner's dilemma. They also present sufficient conditions under which the first-best involves asymmetric treatment of the two players. However, in contrast to us, in the first-best either both receivers are uninformed or both receive some information. Some other related papers are mentioned below.

2. Basic Cournot duopoly

2.1. Model

We consider a symmetric Cournot duopoly with affine inverse demand, in which the intercept is a random variable θ that takes finitely many values in a set $\Theta \subseteq \mathbb{R}_+$. Let $\theta_L \equiv \min \Theta$ and $\theta_H \equiv \max \Theta$. In stage 0, θ is realized but is not observed by the duopolists, agents 1 and 2. In stage 1, each agent i receives a private message $s_i(\theta)$ that satisfies $\theta \in s_i(\theta) \subseteq \Theta$. That is, every agent i is endowed with a partition S_i of Θ and is told which element $s_i(\theta)$ in his partition contains the true state θ. The two partitions, (S_1, S_2), define the *information structure* of the game. In stage 2, the duopolists simultaneously and noncooperatively choose a (positive) quantity to produce. The goods are substitutes and marginal costs are constant.

If the duopolists produce quantities x and y in $\mathbb{R}_+$, they get the payoffs[3]

$$u_1((x,y),\theta) = x\,[\theta - (x+y)]$$
$$u_2((x,y),\theta) = y\,[\theta - (x+y)]\,.$$

We are looking for an information structure that is optimal in the sense that it maximizes the ex-ante expected sum of the duopolists' payoffs. Let

$$\begin{aligned} v((x,y),\theta) &\equiv u_1((x,y),\theta) + u_2((x,y),\theta) \\ &= (x+y)\,[\theta - (x+y)]\,. \end{aligned} \tag{1}$$

$v((x,y),\theta)$ may be interpreted as the payoff of a benevolent planner, who privately observes the value of θ and sends a private message to every duopolist at stage 1. With this interpretation, the duopolists play a Bayesian game in which private information is generated by the planner's messages. Agent 1 chooses $x(s_1(\theta)) = x(\theta)$, $s_1(\theta)$-measurable and similarly, agent 2 chooses $y(s_2(\theta)) = y(\theta)$, $s_2(\theta)$-measurable. Since the duopolists interact noncooperatively, $x(\theta)$ and $y(\theta)$ must be best responses to each other, so that $x(\theta)$ maximizes

$$\mathbb{E}[u_1(x(\theta), y(\theta), \theta) \mid s_1(\theta)] = x(\theta)\,[\mathbb{E}[\theta - y(\theta) \mid s_1(\theta)] - x(\theta)]$$

which describes a parabola with roots 0 and $\mathbb{E}[\theta - y(\theta) \mid s_1(\theta)]$. If $\mathbb{E}[\theta - y(\theta) \mid s_1(\theta)] \le 0$, the max over $\mathbb{R}_+$ is 0. Agent 2's equilibrium condition is similar.

We will first assume that the distribution of θ cannot put much weight on relatively high values, in the sense that

$$\mathbb{E}(\theta) < 3\theta_L. \tag{A}$$

Assumption (A), which is always satisfied if $\theta_H < 3\theta_L$, is equivalent to $\theta - \frac{1}{3}\mathbb{E}(\theta) > 0$ for every θ. It says that an agent who knows the state θ and conjectures that the other agent will choose $\frac{1}{3}\mathbb{E}(\theta)$ produces a positive quantity.

2.2. Optimal information structure

An information structure $\{S_1, S_2\}$ is optimal if it maximizes

$$\begin{aligned} \mathbb{E}\,[v((x,y),\theta)] = \mathbb{E}\,[&\{x(s_1(\theta)) + y(s_2(\theta))\} \\ &\times \{\theta - x(s_1(\theta)) - y(s_2(\theta))\}] \end{aligned} \tag{2}$$

where

$$x(s_1(\theta)) = \max\left\{0, \frac{1}{2}\mathbb{E}[\theta - y(s_2(\theta)) \mid s_1(\theta)]\right\} \tag{3}$$

and similarly for agent 2.

Before state θ is realized, the planner commits to a rule that decides which pair of private messages he will send at every state of nature. The planner's commitment strategy maximizes his ex-ante expected payoff, taking into account the second-stage game between the agents.

The next result identifies the optimal information structure.

Proposition 1. *In the basic Cournot duopoly, if assumption (A) holds, an optimal information structure consists of revealing the state of nature to one duopolist and transmitting no information to the other one.*

Proof. Let

$$\delta(\theta) \equiv x(s_1(\theta)) + y(s_2(\theta)) - \frac{\theta}{2}.$$

Then the objective function (2) may be rewritten as

$$\mathbb{E}\left[\left(\frac{\theta}{2} + \delta(\theta)\right)\left(\frac{\theta}{2} - \delta(\theta)\right)\right] = \mathbb{E}\left[\left(\frac{\theta}{2}\right)^2 - \delta^2(\theta)\right].$$

In the rest of the proof, we simply write x for $x(\theta)$ and y for $y(\theta)$.

Step 1: We show that $\mathbb{E}[\delta(\theta)] \ge \frac{1}{6}\mathbb{E}(\theta)$. We deduce from the above expression (3) for x and the similar expression for y that $\mathbb{E}(x) \ge \frac{1}{2}[\mathbb{E}(\theta) - \mathbb{E}(y)]$ and $\mathbb{E}(y) \ge \frac{1}{2}[\mathbb{E}(\theta) - \mathbb{E}(x)]$. Hence $\mathbb{E}(x+y) \ge \frac{2}{3}\mathbb{E}(\theta)$ and $\mathbb{E}[\delta(\theta)] \ge \frac{1}{6}\mathbb{E}(\theta)$.

Step 2: By Jensen's inequality, $[\mathbb{E}(\delta(\theta))]^2 \le \mathbb{E}(\delta^2(\theta))$. By step 1, this implies that $\mathbb{E}(\delta^2(\theta)) \ge [\frac{\mathbb{E}(\theta)}{6}]^2$. Thus, the objective function (2) satisfies

$$\begin{aligned} \mathbb{E}\left[\left(\frac{\theta}{2}\right)^2 - \delta^2(\theta)\right] &\le \mathbb{E}\left(\left(\frac{\theta}{2}\right)^2\right) - \left(\frac{\mathbb{E}(\theta)}{6}\right)^2 \\ &= \frac{1}{4}\mathbb{E}(\theta^2) - \frac{1}{36}[\mathbb{E}(\theta)]^2. \end{aligned}$$

Step 3: We show that the lower bound on $\mathbb{E}(\delta^2(\theta))$ is achieved when one agent is fully informed and the other agent is not informed at all. Suppose indeed that this happens: the informed agent chooses $\frac{1}{2}\theta - \frac{1}{6}\mathbb{E}(\theta)$, which is ≥ 0 for every θ iff $\frac{1}{3}\mathbb{E}(\theta) \le \theta_L$ (namely, assumption (A)), while the uninformed one chooses $\frac{1}{3}\mathbb{E}(\theta) \ge 0$. It follows that the ex-ante expected sum of the

[2] For example, signals are public in Hagenbach et al. (2014) and Lukyanov and Su (2014); they are not in Perry et al. (2014).

[3] As pointed out by Einy et al. (2010), as soon as there is uncertainty in a Cournot duopoly, possibly negative prices have an impact on existence of an equilibrium. We take the payoff functions seriously, namely, implicitly allow negative prices, but insist on positive quantities. Our interpretation is that, in the case of overproduction (i.e., $x + y > \theta$), the agents face a loss, *as if* the price was negative.

duopolists' payoffs is:

$$\mathbb{E}\left[(x+y)(\theta-(x+y))\right] = \mathbb{E}\left[\left(\frac{1}{2}\theta+\frac{1}{6}\mathbb{E}(\theta)\right)\left(\frac{1}{2}\theta-\frac{1}{6}\mathbb{E}(\theta)\right)\right] = \frac{1}{4}\mathbb{E}(\theta^2)-\frac{1}{36}[\mathbb{E}(\theta)]^2. \quad \blacksquare$$

The planner's first-best is achieved with a total output of $\frac{1}{2}\theta$ in state θ, which generates an ex-ante payoff of $\frac{1}{4}\mathbb{E}(\theta^2)$. Proposition 1 shows that by revealing θ to one agent and nothing to the other, the planner manages to minimize $\mathbb{E}(\delta^2(\theta)) = \mathbb{E}((x+y-\frac{1}{2}\theta)^2)$, while limiting expected production $\mathbb{E}(x+y)$ to exactly $\frac{2}{3}\mathbb{E}(\theta)$.

3. Extensions

This section examines whether Proposition 1 extends to variations of our basic environment.

3.1. Expected intercept

We first show that Proposition 1 may not hold when assumption (A) is violated, i.e., when $\mathbb{E}(\theta) > 3\theta_L$. Suppose there are two states, i.e., $\Theta = \{\theta_L, \theta_H\}$, and that the probability of θ_H is $q \in (0,1)$. Consider two information structures: only one duopolist knows θ (as in Proposition 1) or both know. Let $\mathbb{E}_1(q)$ (resp., $\mathbb{E}_2(q)$) denote the ex-ante expected sum of the duopolists' payoffs in the former (resp., latter) case. Then

$$\mathbb{E}_2(q) = \frac{2}{9}\mathbb{E}(\theta^2) = \frac{2}{9}\left[\theta_L^2 + q(\theta_H^2-\theta_L^2)\right].$$

Let us compute $\mathbb{E}_1(q)$. When only agent 1 knows θ, he maximizes $x(\theta-(x+y))$ over $\mathbb{R}_+$; hence $x(\theta) = \max\left\{0, \frac{1}{2}(\theta-y)\right\}$. In particular, $x(\theta) \le \frac{1}{2}\theta$. Agent 2 maximizes $y[\mathbb{E}(\theta)-(\mathbb{E}(x)+y)]$. Given that $\mathbb{E}(x) \le \frac{1}{2}\mathbb{E}(\theta)$,

$$y = \frac{1}{2}[\mathbb{E}(\theta)-\mathbb{E}(x)] \in \left[0, \frac{1}{2}\theta_H\right]$$

but one cannot exclude that $y > \theta_L$. If this happens, $x_L = 0$, $x_H \equiv x_H(y) = \frac{1}{2}(\theta_H - y)$, $\mathbb{E}(x) = \frac{q}{2}(\theta_H-y)$ and

$$y \equiv y(q) = \frac{2(1-q)\theta_L + q\theta_H}{4-q}.$$

It turns out that $y(q) > \theta_L \Leftrightarrow \mathbb{E}(\theta) > 3\theta_L$ ($\Leftrightarrow q > \frac{2\theta_L}{\theta_H-\theta_L}$), so that if $q > \frac{2\theta_L}{\theta_H-\theta_L}$, $x = \{x_L, x_H\} = \{0, x_H(y(q))\}$ and $y(q)$ are indeed in equilibrium.[4] In this equilibrium, expected utility maximization induces uninformed agent 2 to overproduce in the low state and informed agent 1 best replies by not producing anything in that state.

Finally, when $q > \frac{2\theta_L}{\theta_H-\theta_L}$,

$$\mathbb{E}_1(q) = (1-q)y(\theta_L-y) + q(y+x_H)(\theta_H-(y+x_H)) = (1-q)y(\theta_L-y) + \frac{1}{4}q(\theta_H^2-y^2).$$

Given the previous expressions, one can find parameters values such that $\mathbb{E}_2(q) > \mathbb{E}_1(q)$ implying that the ex-ante expected sum of the duopolists' payoffs is higher when they are both informed than when only one of them is informed (take, e.g., $\theta_L = 1$, $\theta_H = 20$ and $q = 0.9$; then $\mathbb{E}_1(q) = 79.38$ and $\mathbb{E}_2(q) = 80.02$).

3.2. Marginal cost

Suppose that instead of constant marginal costs the cost function was quadratic (this preserves the linearity of best responses). The payoff functions become

$$u_1((x,y),\theta) = x[\theta-(x+y)] - cx^2$$
$$u_2((x,y),\theta) = y[\theta-(x+y)] - cy^2$$

where $c \in \mathbb{R}_+$, while assumption (A) becomes

$$\mathbb{E}(\theta) < (3+2c)\theta_L. \qquad (A')$$

All other assumptions of Section 2.1 are maintained. By adopting the same notation as above, one first computes the ex-ante expected sum $\mathbb{E}_2$ of the duopolists' payoffs when they are both informed of the state:

$$\mathbb{E}_2 = \frac{2(1+c)}{(3+2c)^2}\mathbb{E}(\theta^2).$$

As a benchmark, a perfectly informed monopolist gets the ex-ante expected payoff $\mathbb{E}_M$ defined by

$$\mathbb{E}_M = \frac{1}{4(1+c)}\mathbb{E}(\theta^2).$$

Assuming (A′), when agent 1 knows θ and agent 2 is uninformed, the equilibrium is described by

$$x \equiv x(\theta) = \frac{1}{2(1+c)}\left[\theta - \frac{\mathbb{E}(\theta)}{3+2c}\right]$$
$$y = \frac{\mathbb{E}(\theta)}{3+2c}$$

and one computes that

$$\mathbb{E}_1 = \mathbb{E}_M + \alpha[\mathbb{E}(\theta)]^2$$

where

$$\alpha \equiv \frac{4c^2+4c-1}{4(1+c)(3+2c)^2} \ge 0 \Leftrightarrow c \ge \frac{\sqrt{8}-2}{4}.$$

In addition,

$$\mathbb{E}_2 = \mathbb{E}_M + \alpha\mathbb{E}(\theta^2).$$

Hence it is better to inform both agents ($\mathbb{E}_2 \ge \mathbb{E}_1$) as soon as $\alpha \ge 0$. Proposition 1 is no longer true for "large" quadratic costs.

Let $\mathbb{E}_0$ be the ex-ante expected sum of the duopolists' payoffs when none of them gets any information:

$$\mathbb{E}_0 = \frac{2(1+c)}{(3+2c)^2}[\mathbb{E}(\theta)]^2.$$

Note that $\mathbb{E}_0 \le \mathbb{E}_1$ and $\mathbb{E}_0 \le \mathbb{E}_2$. If $\alpha \le 0$ ("small" c, e.g., $c = 0$, as in Section 2), $\mathbb{E}_0 \le \mathbb{E}_2 \le \mathbb{E}_1 \le \mathbb{E}_M$. If $\alpha \ge 0$, $\mathbb{E}_M \le \mathbb{E}_1 \le \mathbb{E}_2$, but $\mathbb{E}_0$ may be below or above $\mathbb{E}_M$.

The previous example shows that our result does not go through in a class of popular quadratic payoff functions considered, e.g., in Bergemann and Morris (2013). That paper points out (pp. 1280–1281) that this is also true of some early results in the literature on information sharing in oligopoly.

3.3. Substitute/complementary goods

Suppose the duopolists sold complementary goods such that their payoff functions are

$$u_1((x,y),\theta) = x[\theta - x + y]$$
$$u_2((x,y),\theta) = y[\theta - y + x].$$

Agent 1's equilibrium condition becomes

$$x(s_1(\theta)) = \frac{1}{2}\mathbb{E}[\theta + y(s_2(\theta)) \mid s_1(\theta)]$$

and similarly for agent 2. The sum of the agents' payoffs is thus

$$u_1((x,y),\theta) + u_2((x,y),\theta) = \theta(x+y) - (x-y)^2.$$

[4] One also checks that this is the only equilibrium when $q > \frac{2\theta_L}{\theta_H-\theta_L}$.

If both duopolists are fully informed, then $x(\theta) = y(\theta) = \theta$ so that the ex-ante expected sum of their payoffs is $\mathbb{E}_2 = 2\mathbb{E}(\theta^2)$. If agent 1 is fully informed and agent 2 is uninformed, then $x(\theta) = \frac{1}{2}[\theta + \mathbb{E}(\theta)]$ and $y = \mathbb{E}(\theta)$; the ex-ante expected sum of their payoffs is $\mathbb{E}_1 = \frac{1}{4}\mathbb{E}(\theta^2) + \frac{7}{4}[E(\theta)]^2$. Hence it is always true that $\mathbb{E}_2 \geq \mathbb{E}_1$ and Proposition 1 does not hold.[5]

3.4. Oligopoly

Assume that there are n oligopolists. Every agent $i = 1, \ldots, n$ chooses a quantity x_i in $\mathbb{R}_+$; let $\mathbf{x} = (x_i)_{1\leq i\leq n}$; the utility functions are

$$u_i(\mathbf{x}, \theta) = x_i \left[\theta - \sum_{j=1}^{n} x_j\right].$$

Given an information structure $\{S_i\}_{1\leq i\leq n}$, exactly as in (3), the equilibrium quantity chosen by agent $i = 1, \ldots, n$ satisfies

$$x(s_i(\theta)) = \max\left\{0, \frac{1}{2}\mathbb{E}\left[\theta - \sum_{j=1}^{n} x_j(s_j(\theta)) \mid s_1(\theta)\right]\right\}. \quad (4)$$

By proceeding as in the proof of Proposition 1, the ex-ante expected sum of the oligopolists' payoffs can be written as

$$\mathbb{E}\left[\left(\frac{\theta}{2}\right)^2 - \delta^2(\theta)\right] \quad (5)$$

where

$$\delta(\theta) = \sum_{j=1}^{n} x_j - \frac{\theta}{2}.$$

The proof of Proposition 1 extends as follows:

Step 1: Under (4),

$$\mathbb{E}[\delta(\theta)] \geq \frac{n-1}{2(n+1)}\mathbb{E}(\theta).$$

Step 2: By Jensen's inequality, the objective function (5) satisfies

$$\mathbb{E}\left[\left(\frac{\theta}{2}\right)^2 - \delta^2(\theta)\right] \leq \frac{1}{4}\left[\mathbb{E}(\theta^2) - \left(\frac{n-1}{n+1}\right)^2[\mathbb{E}(\theta)]^2\right]. \quad (6)$$

Step 3: Assume that $\mathbb{E}(\theta) \leq \frac{n+1}{n-1}\theta_L$ (extending assumption (A)). If only one agent, say agent 1, is fully informed while each of the others is uninformed, the equilibrium quantities are

$$x_1(\theta) = \frac{1}{2}\left[\theta - \frac{n-1}{n+1}\mathbb{E}(\theta)\right], \qquad x_j = \frac{\mathbb{E}(\theta)}{n+1}, \quad j \neq 1$$

which achieve the upper bound in (6).

Note, however, that as $n \to \infty$ the assumption $\mathbb{E}(\theta) \leq \frac{n+1}{n-1}\theta_L$ becomes more demanding.

3.5. Planner's constraints and objective

If the planner must send the *same* message $s(\theta)$ to *both* duopolists, the maximal sum of their expected payoffs is achieved by fully informing both of them (and is then $\frac{2}{9}\mathbb{E}(\theta^2)$).

If the planner's objective is to maximize the expected sum of *outputs*, such that each firm produces a positive amount, then (3) implies that their expected outputs are equal, $\mathbb{E}(x) = \mathbb{E}(y) = \frac{1}{3}\mathbb{E}(\theta)$, whatever the information structure (see step 1 of the proof of Proposition 1).[6] However, under assumption (A), the total expected surplus, $\mathbb{E}\left(\theta(x+y) - \frac{1}{2}(x+y)^2\right)$, is larger when the duopolists are both informed than when only one of them is. Focusing on this specific information structures, Einy et al. (2013) and Warneryd (2003) reach similar conclusions (in terms of effort and surplus) for Tullock's contests.

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[5] In fact, for the payoff functions of this Section 3.3, the optimal information structure consists of fully informing both agents (Antoine Salomon, private communication).

[6] In the example of Section 3.1, a larger expected total output is achieved by informing only one duopolists, but with overproduction of the uninformed one in the low state.