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Strategic disclosure of feasible options [☆]

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ABSTRACT

The paper illustrates how people who need to make a joint decision may have an incentive to withhold information regarding the existence of Pareto improving options. The resulting level of inefficiency varies with the way compromises are reached when the parties have to choose among multiple options. Various reasonable compromise rules can be ranked unequivocally, and a rule resulting in a minimal level of inefficiency is identified. Qualitative results extend to sequential disclosure. Enforcing a hard deadline for disclosure may be welfare improving in some circumstances.

1. Introduction

Suppose some individuals (e.g. members of an organization, or bargainers) hold private and verifiable information regarding the availability of various feasible options. What are their incentives to disclose such information? We argue that withholding one's information may sometimes be preferable even if it is Pareto improving. Indeed, one must take into account the possibility that another participant may disclose an alternative action that is feasible and relatively more advantageous to oneself. We study how incentives to disclose vary with the way compromises are reached when parties must choose among multiple feasible options.

Such considerations are relevant, for instance, when a committee decides on hiring a candidate while it is not commonly known which individuals are available and/or suitable for the task (e.g. members of a research department hiring a junior colleague). Another example is that of a husband and a wife, who need to decide which house to buy, or which city/state/country to move to, or which school to send their kids to. Both sides need to identify feasible alternatives (e.g., which houses are for sale, which locations have relevant job openings, which schools have open slots), and the two may have conflicting interests regarding the choice (e.g., each may want a house closer to his/her workplace, each may want to move to a location with better career opportunities or closer to one's family, each may have a different preference on public versus private education).

We start by studying the simplest model capturing the relevant strategic considerations we aim to illustrate. Two individuals are each privately aware of the feasibility of an option resulting in utility pairs that fall on the straight line joining $(1,0)$ to $(0,1)$. Later (see Section 6), we indicate how our results extend to more general frameworks, including situations

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where individuals may be unaware of any feasible action with positive probability, situations where individuals may be aware of more than one feasible action, and situations where feasible options result in utility pairs that fall on a symmetric curve that need not be straight.[2]

In the static disclosure game, participants must decide simultaneously whether to disclose the feasible option they are aware of. Depending on the strategy they follow, and the hard evidence they hold, either zero, one or two feasible options will be disclosed. The final outcome and resulting utilities then depend on how collective decisions are made. Various procedures come to mind. Perhaps the simplest is to choose at random with the help of a fair coin. More generally, collective decisions could result from the implementation of given bargaining protocols, or the application of basic fairness principles. Our analysis accommodates diverse ways of reaching compromises. Formally, a compromise rule associates with every set of disclosed options, a lottery over these options, or more precisely the pair of expected utilities associated to this lottery. It is "regular" if it satisfies basic properties of anonymity, ex-post efficiency, and monotonicity (see definitions in Section 2). Proposition 1 provides a characterization of the (unique) symmetric Bayesian Nash equilibrium of the disclosure game given any regular compromise rule.

Our second main result is normative in nature. Are some regular rules unequivocally superior to others in terms of the efficiency level they induce in the disclosure game? We define a partial order on compromise rules, and prove that the efficiency level is increasing with that order. It also allows to identify an optimal regular rule, which happens to coincide with the Nash (1950) solution in the special case where utility pairs associated with collective decisions fall on the straight line joining on $(1, 0)$ and $(0, 1)$.

It is tempting to think that inefficiency is driven by the fact that there is a single opportunity to speak. Wouldn't individuals give in, and disclose their information, right before they are about to settle on the Pareto inferior status quo because no option has been disclosed? Not necessarily: at any point in time, individual i prefers j to give in first when there is a high likelihood that j is aware of an option that is relatively more advantageous to i. To address this question we analyze two variants of a dynamic disclosure game. Both variants exhibit inefficiency in the form of delay. In the first variant, where a choice is made as soon as one option is disclosed, the partial ranking we identified in our analysis of the static game remains relevant as information gets disclosed no later when using a superior compromise rule. In particular, the compromise rule that was optimal for the static game remains optimal for the dynamic game as well. In the second variant, where each player can react to a disclosure of the other party, any regular compromise rule induces the *same* (perfect Bayesian Nash) equilibrium probability of disclosure, which is *lower* than the equilibrium probability when players cannot respond to disclosure.

Our analysis allows us to investigate whether enforcing deadlines enhances mutual disclosure. The static game exhibits an extreme deadline as players must disclose by some prespecified date. The first variant of the dynamic game exhibits a more flexible deadline in the sense that a player must disclose no later than his opponent. The second variant of the dynamic game essentially has no deadline. While a flexible deadline is preferred to no deadline in terms of delay, we cannot make a general welfare comparisons between the static and dynamic games. Instead, we provide a robust example where enforcing a hard deadline, namely to speak once or never thereafter, is preferable to letting individuals speak at any point in time. This illustrates that limiting the opportunity to disclose may have a positive impact on welfare.

Related literature Our paper fits into the literature on information revelation in committees (see e.g. Hao and Suen, 2009 for a survey), as well as the literature on expert advices (see e.g. Sobel, 2010 for a survey) if one interprets the compromise rule as capturing the decision of a fictitious third party. Milgrom and Roberts (1986) is the closest model within this literature as it shares two important features with our setting, the presence of multiple "senders" (the two informed individuals) and of verifiable information (a problem of disclosure instead of cheap talk). Some key differences, though, lead to distinct results. Competition among multiple senders leads to first-best efficiency in their model, but not in ours because information is incomplete. An individual does not know indeed the option that his partner is aware of. Classic results on information unraveling do not apply either because information is about the feasibility of an option, not about a state of nature that impacts preferences. If an individual knows that his opponent is withholding information regarding the feasibility of some option, there is nothing he or the third party can do without knowing what that collective action is.

In a classic paper, Kalai and Samet (1985) study a non-cooperative destruction game that precedes a bargaining stage. Opportunities at the bargaining stage are captured in their model by utility possibility sets. In the prebargaining stage, individuals simultaneously choose which utility pairs (if any) to take away from that set. A proportional solution for the bargaining stage associates to each utility possibility set the utility pair in it that is as high as possible on a given straight line with positive slope. When payoffs associated to disclosed options fall on the line joining $(1, 0)$ to $(0, 1)$, as in most of our paper, a proportional solution picks $(\alpha, 1-\alpha)$ for a given $\alpha \in [0, 1]$ if such a utility pair is feasible, and $(0, 0)$ otherwise. Kalai and Samet observe that no destruction is a dominant strategy for both bargainers when bargaining outcomes coincide with a fixed proportional solution.[3] While our game involves disclosure instead of destruction, and is of incomplete instead

[2] Extensions to asymmetric problems, problems involving more than two individuals, and problems with feasible actions that result in Pareto comparable utility profiles are left for future work.

[3] Kalai and Samet also establish a partial converse result when utility possibility sets are comprehensive: (a) no destruction is a dominant strategy in all problems only if the solution is monotone (a different notion of monotonicity than the one we consider below), and (b) a solution is monotone, (weakly) Pareto efficient, homogenous and strictly individually rational if and only if it is proportional (see also Kalai, 1977).

of complete information, a similar result applies in our framework: compromising via a given proportional solution makes disclosure dominant in all circumstances. Proportional compromise rules are not regular (and thus not covered by our analysis) because they are ex-post inefficient. Rules that are ex-post inefficient are unlikely to be followed, as they are not renegotiation-proof. For instance, the proportional rule selects the status quo when only one individual reveals his type in the disclosure game,[4] despite the fact that a Pareto superior option is available. A similar problem occurs when both individuals disclose feasible options that deliver utility pairs that are both above or both below the straight line defining the proportional solution. Beyond the difficulty associated with a lack of renegotiation-proofness, we show in the supplementary online appendix that the efficiency loss associated with proportional rules (due to inefficiency at the compromise stage) is larger than the efficiency loss of our optimal ex-post efficient rule (due to inefficiency at the disclosure stage), except in rare circumstances where individuals face a high likelihood of knowing an option that is “unfavorable” (see Definition 1) to them.

Frankel (1998) provides a first analysis of individual incentives to identify feasible options, a problem of acquisition instead of transmission of information. For instance, in one of the models he studies, each bargainer i simultaneously chooses a probability p_i and an interval $I_i \subseteq [\frac{1}{2}, 1]$, incurring a cost, which increases with p_i and decreases with the length of I_i. With probability p_i bargainer i gets an “idea”, which gives him a payoff of x_i, where x_i is drawn from a uniform distribution on I_i. With probability $1 - p_i$, bargainer i gets no idea. The realizations of the two bargainers are then revealed (they have no choice on the matter) and a coin is tossed to select one of the realized ideas (if only one idea was realized, it is selected with certainty), and if no idea was realized, both get zero. Frankel shows that in equilibrium, the choices of $(p_i, I_i)_{i=1,2}$ can be either excessive or suboptimal. Determining how different compromise rules affect individual incentives to acquire costly information regarding the feasibility of options, and building on our model by adding costly search as a preliminary stage to strategic disclosure are both interesting questions for future research.

While some features of the disclosure games we study are reminiscent of games where players have to make concessions, there are important differences between the two. While there is no agreed-upon definition of a concession game, some consider games in which every period a player decides how much of the pie to give up to the other player (as in Compte and Jehiel, 2004), while others (as in Ordover and Rubinstein, 1986) consider games where each player needs to decide up to some deadline whether to accept the other player’s favorite outcome. In contrast, in our framework a player who “acts” discloses an option, which may be strictly *better* for him than the option known to the other player. In this sense, disclosing information is not analogous to conceding since the disclosing player can *gain* (at the expense of the other player) from disclosing. In addition, whether or not a player i discloses his option, his payoff will depend on player j’s type whenever j discloses.

Finally, the dynamic disclosure game, presented in Section 5, is related to the analysis of the war of attrition and related games of costly waiting in continuous time and with asymmetric information (see e.g., Gul and Lundholm, 1995; Ponsati and Sákovics, 1995; Bulow and Klemperer, 1999). The key difference between these games and ours is that each player’s payoff depends on the other player’s type, either directly when one’s opponent has disclosed an option, or via the compromise rule when both have disclosed at the same time (as may happen in the unique symmetric equilibrium of our game). In addition, the war of attrition has oftentimes been analyzed as an all-pay auction where the players’ preferences are quasi-linear in the cost of delay (see, e.g., the “generalized” war of attrition by Bulow and Klemperer, 1999), which is not the case in our framework. These differences imply that standard techniques used to solve the above waiting games do not apply to the present context.

2. Benchmark model

Two individuals face a large set of feasible options, but do not know which are actually feasible and/or what the associated payoffs are. Each has learned, either by chance or as a result of active search,[5] about the feasibility of a single option, represented in the space of utilities as a pair of non-negative real numbers. As a starting point, we assume that these payoffs fall on the straight line joining $(1, 0)$ to $(0, 1)$, with $(0, 0)$ determining the reference payoffs that prevail when no option is chosen. We show in Section 6 that our analysis extends to more general sets, and to problems where an individual may be aware of multiple actions or none at all.

To simplify the exposition we define an individual’s type to be his utility from the payoff pair he is aware of. We let x denote the first individual’s type (i.e., he is aware of the payoff pair $(x, 1 - x)$), and we let y denote the other individual’s type (i.e., he is aware of $(1 - y, y)$).

An individual does not know what utility pair his opponent is aware of. Beliefs are captured by a common density f on $[0, 1]$ with full support. Consider for instance the case of two parties with conflicting objectives who need to agree on a person to hire. While parties may know the distribution of the potential candidates’ characteristics in the population, the characteristics of any given candidate and whether he or she is interested in being considered for the position are not self-evident.

[4] Except in the zero probability event where the disclosed option results in utilities with the right proportions.

[5] In this latter case, our work can serve as a building block of a more elaborate model designed to understand incentives to search before the disclosure stage.

The game starts with the *disclosure stage*. At that point, individuals decide independently whether to disclose the feasibility of the collective action they are aware of.[6] We assume that types are verifiable once disclosed, and hence an individual cannot report anything else than what he knows. The benchmark model may be viewed as a situation in which the two parties need to submit hard evidence on a feasible option (from which the payoffs can be inferred) until some prespecified (and rigid) deadline, and submissions will be reviewed only at the deadline (i.e., submitting before the deadline has no effect, and evidence submitted after the deadline is not accepted).

The game ends with the *compromise stage*, where individuals decide which option to pick. Only those actions for which there is verifiable evidence attesting to their feasibility (and from which the payoffs can be inferred) can be selected. Going back to our hiring example, members of the hiring committee can only choose among a list of candidates that is presented to them.

Including the possibility of making no choice, there can be at most three alternatives to choose from depending on the number of options that have been disclosed. We study problems where monetary compensations are not available, or deemed inappropriate. Compromises can thus be reached only by using *lotteries* over the disclosed options.[7] The details of the underlying process determining how this lottery depends on the set of available options does not matter for our analysis. All what matters is how the final expected payoffs vary, which will be captured by a *compromise rule* $r : ([0,1] \times \{\emptyset\}) \times ([0,1] \times \{\emptyset\}) \to \mathbb{R}^2_+$. We assume that $r(\emptyset, \emptyset) = (0,0)$, i.e., if no option was disclosed then the only available alternative – do nothing – is selected. The compromise rule can be a reduced-form of the equilibrium outcome of some explicit bargaining protocol, or of the outcome associated with some fairness criteria, or of some previously agreed-upon standard.

Our analysis applies to any compromise rule r that satisfies the following three regularity properties. A *regular* compromise rule is

1. *ex-post efficient*, meaning that it selects utility pairs that are Pareto efficient given the feasible options that have been disclosed: for all $x, y \in [0,1]$, $r(x, \emptyset) = (x, 1-x)$, $r(\emptyset, y) = (1-y, y)$, and there exists $\alpha \in [0,1]$ such that $r(x, y) = \alpha(x, 1-x) + (1-\alpha)(1-y, y)$.
2. *anonymous*, meaning that the individuals' identity is irrelevant for determining the compromise: $r_i(x, y) = r_j(y, x)$.
3. *monotone*, in the sense of giving a larger utility to an individual when disclosed actions are more favorable to him: $x' \geq x, y' \leq y \Rightarrow r_1(x', y') \geq r_1(x, y)$ and $r_2(x', y') \leq r_2(x, y)$, for all x, x', y, and y' in $[0,1]$.

The regularity conditions are meant to capture common features of prevalent collective decision processes. The first part of our analysis will be positive, characterizing symmetric Bayesian Nash equilibria of the disclosure game for a given regular compromise rule, and computing their associated level of efficiency. For this part, we highlight that our regularity conditions are met by all classic collective decision processes discussed in the literature (non-cooperative and axiomatic bargaining, as well as social choice). The second part of our analysis is normative, characterizing a regular compromise rule that systematically minimizes the level of inefficiency at the disclosure stage. This could be viewed as a constrained mechanism design exercise. Even if they might be ready to change to some extent the way compromises are reached in order to improve efficiency at the disclosure stage, they are not ready to give up on these basic properties that are of primary importance to them.

Example 1. Ex-post efficiency fully determines which option to implement when only one has been disclosed. Compromise rules thus vary only in how the final outcome is determined when two collective decisions have been disclosed. The simplest way to reach a compromise in such circumstances is to decide which action to implement with the help of a fair coin.[8] The coin-flip rule is thus defined by

$$r_{CF}(x, y) = \left(\frac{x + (1-y)}{2}, \frac{(1-x) + y}{2}\right)$$

for all $x, y \in [0,1]$.

A second classic way to resolve conflict is captured by Nash's (1950) bargaining solution. It is obtained by maximizing the product of the two individuals' utilities. In our benchmark model, this rule picks the lottery that brings the individuals' utilities as close as possible to $(1/2, 1/2)$. Formally:

$$r_N(x, y) = \begin{cases} (\max\{x, 1-y\}, 1 - \max\{x, 1-y\}) & \text{if } \max\{x, 1-y\} \leq \frac{1}{2} \\ (\min\{x, 1-y\}, 1 - \min\{x, 1-y\}) & \text{if } \min\{x, 1-y\} \geq \frac{1}{2} \\ (\frac{1}{2}, \frac{1}{2}) & \text{otherwise.} \end{cases}$$

[6] We assume that no exogenously imposed sequence of moves can be enforced. In Section 5 we endogenize the sequence of moves by letting individuals decide if and when to disclose.

[7] Random mechanisms are ubiquitous means for achieving procedural fairness in situations where parties disagree on which outcome should be implemented. For example, randomization is used in procedures for selecting an arbitrator (see de Clippel et al., 2014) and in allocating goods among a group of individuals (see, e.g., Budish et al., 2013 and the references therein).

[8] This rule coincides to Raiffa's (1957, Section 6.7) solution in bargaining theory.

A third bargaining solution that is often discussed in the literature was proposed by Kalai and Smorodinsky (1975). This rule also appears in social choice under the name of relative egalitarianism. Indeed, it picks the Pareto efficient utility profile that equalizes the individuals' utility gains, relative to the maximal utility they could achieve. Formally:

$$r_{KS}(x,y) = \left(\frac{\max(x,1-y)}{\max(x,1-y)+\max(1-x,y)}, \frac{\max(1-x,y)}{\max(x,1-y)+\max(1-x,y)}\right).$$

3. Positive analysis of the disclosure stage

A mixed-strategy for individual i in the disclosure stage is a measurable function $\sigma_i : [0,1] \to [0,1]$, where $\sigma_i(x)$ is the probability that type x of individual i announces the option he is aware of. A pair of mixed strategies, one for each individual, forms a *Bayesian Nash equilibrium* (BNE) of the disclosure game if the action it prescribes to each type of each individual is optimal against the opponent's strategy. The BNE is *symmetric* if both individuals follow the same strategy.

Since no feasible option Pareto dominates another, inefficiency can only stem from no disclosure. Hence, the inefficiency induced by a compromise rule in some BNE is equal to the probability of no-disclosure in that equilibrium. A compromise rule induces an efficient equilibrium if it induces an equilibrium where at least one party discloses for sure.

Proposition 1. *The disclosure game admits a unique symmetric BNE, in which each individual discloses his type if and only if it is greater or equal to a strictly positive threshold*

$$\theta = \sup\{x \in [0, \tfrac{1}{2}] \mid xF(x) + \int_{y=x}^{1} (r_1(x,y) - (1-y))f(y)dy < 0\}.\text{[9]} \tag{1}$$

The associated equilibrium outcome is thus inefficient.

The proof relies on properties of the function determining the expected net gain of revealing over withholding one's type given the opponent's strategy. For individual 1 this is given by

$$ENG_1(x,\sigma_2) = x\int_{y=0}^{1} (1-\sigma_2(y))f(y)dy + \int_{y=0}^{1} \sigma_2(y)[r_1(x,y) - (1-y)]f(y)dy, \tag{2}$$

for each type $x \in [0,1]$ and each strategy σ_2 (ENG for is defined analogously for the second individual). It is not difficult to check (see Lemma 1 in Appendix A) that ENG increases with types (independently of the other individual's strategy), and strictly decreases as the other individual becomes more likely to disclose each of his types. As a consequence, BNEs must involve threshold strategies, and equilibrium existence is proved using Brouwer's fixed point theorem.[10]

If the symmetric BNE is the unique BNE (as is the case, for instance, when applying the coin-flip rule with a uniform distribution of types), then it is the only rationalizable strategy profile.

Proposition 2. *If the symmetric BNE is the unique BNE, then it is also the unique profile of strategies that survives the iterated elimination of strictly dominated strategies.*

Our analysis focuses on symmetric BNEs. This is natural given that individuals are symmetric (ex-ante) with r being anonymous, and types being drawn from a same distribution. The disclosure game may also have asymmetric BNEs in addition to the unique symmetric one. Inefficiency often prevails at all those equilibria as well. A compromise rule is *symmetric* if the utility pair it selects depends only on the payoffs associated with the feasible options that have been disclosed, and not on the identity of who has disclosed what: $r(x,y) = r(1-y, 1-x)$. This property is met by all classic compromise rules, and in particular all compromise rules provided as examples in this paper.

Proposition 3. *Let r be a regular compromise rule that is continuous and symmetric. If it is more likely to discover collective decisions that are relatively more favorable, in the sense that $f(x) > f(1-x)$, whenever $x \geq 1/2$, then the outcome associated with any BNE of the disclosure game associated with r is inefficient.*

[9] If r is continuous, then the threshold θ is given by the solution to the following simpler equation: $\theta F(\theta) + \int_{y=\theta}^{1}(r_1(\theta,y) - (1-y))f(y)dy = 0$.

[10] Observe that existence is obtained without requiring compromise rules to be continuous.

4. How to promote disclosure?

Inefficiency arises in our setting when no individual reveals his type in the disclosure stage. The expected level of inefficiency is thus equal to the probability of that event, which is the square of the threshold characterized in Eq. (1). The supplementary online appendix contains computations of this threshold for r_{CF}, r_{KS}, and r_N in the case of a uniform distribution of types. The resulting numbers are 0.236, 0.22, and 0.183 respectively. Hence we see that using the Nash solution as a compromise rule is welfare improving compared to Kalai–Smorodinsky, which in turn is preferable to coin-flipping. Can we rank other compromise rules? Are there alternatives that are preferable to r_N? Do results depend on the specific distribution of types? In order to address these questions, we introduce a partial ordering on compromise rules.

Definition 1. An individual's type is *unfavorable* if the payoff that is relevant for him is lower than 1/2. The first individual is in a *weaker position* if $x \leq y$. Similarly, the second individual is in a *weaker position* if $y \leq x$.

Notice that $x \leq y$ is equivalent to $\min\{x, 1-y\} \leq \min\{1-x, y\}$. In other words, the first individual is in a weaker position if the worst payoff he can get, given the disclosed feasible options, is lower than the worst payoff of the other individual.[11]

Definition 2. The compromise rule r' dominates the compromise rule r if r' gives a higher payoff than r to an individual with an unfavorable type who is in a weaker position. Formally, $r' \succeq r$ if $r'_1(x, y) \geq r_1(x, y)$, for all x, y such that $x \leq \min\{1/2, y\}$, and $r'_2(x, y) \geq r_2(x, y)$, for all x, y such that $y \leq \min\{1/2, x\}$.

Proposition 4. *If the compromise rule r' dominates the compromise rule r, then the probability of inefficiency in the symmetric equilibrium of the disclosure game associated with r' is smaller or equal to the probability of inefficiency in the symmetric equilibrium of the disclosure game associated with r.*

Proposition 4 allows us to conclude that the Nash solution is an optimal regular rule in the sense that it induces the highest equilibrium probability of disclosure (in the supplementary online appendix we show that $r_{KS} \succeq r_{CF}$, and hence, the Kalai–Smorodinsky solution is at least as efficient as flipping a coin). For an intuition why the Nash solution is ranked highest according to our ordering, notice that on our restricted domain the Nash solution coincides with the *lexicographic-egalitarian* (LE) solution (Rawls, 1971). When the disagreement point is the origin, the LE solution is computed for any given set of payoff profiles by the following induction: first select the subset of options that maximize the payoff of the worst-off individual, then select a subset of the set of options identified in the first step by maximizing the utility of the next to worst-off individual, and iterate this process until all possibilities for increasing the utility of any individual have been exhausted. Thus, in our disclosure game, the LE solution maximizes the payoff of the player who is in the weaker position whenever only one of the two players has an unfavorable type. For all other combinations of types, the utility pair (1/2, 1/2) is achievable through some lottery when both players disclose, and LE picks it. Hence, LE should be ranked highest given the partial ordering defined above.

Corollary 1. *$r_N \succeq r$, for any regular compromise rule r. Hence the probability of inefficiency in the symmetric equilibrium of the disclosure game associated with any regular compromise rule is larger or equal to the probability of inefficiency in the symmetric equilibrium of the disclosure game associated with the Nash solution.*

A dual to Corollary 1 gives an upper bound on the probability of inefficiency associated with any regular compromise rule. Consider the compromise rule that maximizes the maximum of the two individuals' payoffs,

$$r_{MM}(x, y) = \begin{cases} (x, 1-x) & \text{if } \max\{x, 1-x\} > \max\{1-y, y\} \\ (1-y, y) & \text{if } \max\{x, 1-x\} < \max\{1-y, y\} \\ (1/2, 1/2) & \text{if } x = y \end{cases}$$

(in other words, this solution picks the point that is the furthest from (1/2, 1/2), i.e., it minimizes the product of the individuals' payoffs). It is easy to check that r_{MM} is regular, and that $r \succeq r_{MM}$, for any regular compromise rule r. By Proposition 4, the probability of inefficiency in the symmetric equilibrium of the disclosure game induced by any regular compromise rule is smaller or equal to the probability of inefficiency in the symmetric equilibrium of the game induced by r_{MM}.[12]

We conclude this section by discussing asymmetric BNEs. The distribution f is symmetric if types x and $1-x$ are equally likely: $f(x) = f(1-x)$, for all $x \in [0, 1]$. The Nash solution is optimal for such distributions, not only when considering the more natural symmetric BNEs, but also when considering asymmetric BNEs.

[11] Note that all regular compromise rules are consistent with the concept of weak position, giving a smaller payoff to the individual who is in a weaker position. Indeed, suppose, for instance, that 1 is weaker, i.e. $x \leq y$. Anonymity implies that $r_1(x, x) = 1/2$. Monotonicity implies that $r_1(x, y) \leq 1/2$, and hence $r_1(x, y) \leq r_2(x, y)$.

[12] For instance, the equilibrium strategies' threshold is equal to 0.293 in the case of a uniform distribution on types.

Proposition 5. *Let r be a regular compromise rule that is continuous and symmetric. If f is symmetric as well, then there exists an efficient BNE in the disclosure game induced by r only if r is the Nash solution.*

5. Dynamic disclosure game

In the dynamic disclosure game, individuals decide when to disclose their type, if at all. The compromise rule is applied as soon as at least one option has been disclosed (we also study below the case where individuals have an opportunity to react before the compromise rule is applied). We restrict attention to symmetric pure-strategy Bayesian Nash equilibria. A strategy is a measurable function $\tau : [0,1] \to \mathbb{R}_+ \cup \{\infty\}$, which determines for each type x the time $\tau(x)$ at which to reveal x.[13] Measurability means that the inverse image of any Lebesgue measurable set (in particular any interval) is Lebesgue measurable: $\tau^{-1}(T) = \{x \in [0,1] | \tau(x) \in T\}$ is Lebesgue measurable if T is Lebesgue measurable. It guarantees that an individual's expected utility when his opponent is known to reveal over some given interval of time, is well-defined. Utilities are discounted exponentially over time following a discount factor $\delta < 1$. The outcome when the first individual is of type x, while the other individual is of type y, and they both implement the strategy τ, is x at time $\tau(x)$ if $\tau(x) < \tau(y)$, y at time $\tau(y)$ if $\tau(x) > \tau(y)$, and $r(x, y)$ at time $\tau(x)$ if $\tau(x) = \tau(y)$.[14]

5.1. Positive analysis

The strategy τ is part of a *symmetric Bayesian Nash equilibrium* if, for every type $x \in [0,1]$, the expected net gain of revealing at any time $t \geq 0$ different from $\tau(x)$ is non-positive, where an individual's expected net gain – let's say the first individual to fix ideas – is given by the following formula when $t > \tau(x)$ (a similar formula applies in the other case):

$$\begin{aligned} ENG_1(t \text{ vs. } \tau(x), x) &= x(e^{-\delta t} - e^{-\delta \tau(x)}) \int_{y \in \tau^{-1}(]t,\infty])} f(y)dy \\ &+ \int_{y \in \tau^{-1}(t)} (e^{-\delta t} r_1(x,y) - e^{-\delta \tau(x)} x) f(y)dy \\ &+ \int_{y \in \tau^{-1}(]\tau(x),t[)} (e^{-\delta \tau(y)}(1-y) - e^{-\delta \tau(x)} x) f(y)dy \\ &+ \int_{y \in \tau^{-1}(\tau(x))} e^{-\delta \tau(x)}((1-y) - r_1(x,y)) f(y)dy. \end{aligned}$$

We will need the following additional assumption on r to establish the uniqueness of the symmetric BNE:

$$r_1(x, \frac{1}{2}) < x, \forall x > 1/2, \text{ and } r_1(x, \frac{1}{2}) > x, \forall x < 1/2.^{15} \tag{3}$$

The weak inequality holds for any regular rule. Requiring a strict inequality is a mild additional requirement which is satisfied by all classic rules (Kalai–Smorodinsky, Nash and coin flip), but not the max–max rule (r_{MM}).

Proposition 6. *Let τ^* be the strategy defined as follows:*

$$\tau^*(x) = \begin{cases} 0 & \text{if } x \geq \theta \\ \int_x^\theta \frac{(1-2y)f(y)}{\delta y F(y)} dy & \text{if } x < \theta, \end{cases}$$

where

$$\theta = \sup\{x \in [0,1/2] | \int_{y=x}^{1} (r_1(x,y) - (1-y)) f(y)dy < 0\}. \tag{4}$$

The pair of strategies (τ^*, τ^*) forms a symmetric Bayesian Nash equilibrium of the dynamic disclosure game. If r satisfies condition (3), then it is the unique symmetric BNE of the game.

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13 $\tau(x) = \infty$ means that the individual never discloses when of type x.

14 The dynamic disclosure game is similar to classic wars of attrition, in that both parties incur a cost of delay when neither gives in. Our discussion of the related literature in the Introduction indicates key differences which makes it impossible to apply results from that literature to our disclosure problem.

15 A similar pair of conditions necessarily hold for the second individual as well, as a consequence of the second regularity condition (Anonymity).

To understand the intuition for this result, consider first the threshold for immediate disclosure, as given by (4). Roughly speaking, the type at the threshold is indifferent between disclosing immediately and disclosing a "little" later, say at ε. The potential gain from waiting is that weaker types may disclose more favorable feasible options. The potential loss is that the game may end with the opponent disclosing before him a type which leads to a less favorable outcome. Since ε is sufficiently small, there is no loss from delay (hence, the discount factor does not appear in (4)), and almost all types who disclose after time zero, also disclose after time ε. This latter point explains why the term $xF(x)$ (the payoff from types weaker than x), does not appear in the equation for the dynamic cutoff (although it does appear in Eq. (1) that determines the static cutoff). The cutoff type θ cannot be higher than $\frac{1}{2}$ because the types who disclose immediately would give θ a lower payoff than if he would compromise. On the other hand, θ cannot be too low because there would be an incentive to delay disclosure as the other party's type is likely to be lower than $1-\theta$.

Note that the compromise rule affects only the threshold of types who disclose immediately. It has no effect on the rate at which types lower than θ delay their disclosure. This follows from the fact that the compromise rule affects the outcome whenever there is a mass of types who disclose at the same time. This occurs at time zero where more than half of the types disclose. However, relatively unfavorable types, who delay their disclosure, have no incentive to pool and disclose at the same time. Intuitively, the most unfavorable type within the set of types who pool would have an incentive to delay his disclosure by a "little bit": all the types he pooled with before would now disclose before him, and since their type is lower than $\frac{1}{2}$, he would get a payoff higher than $\frac{1}{2}$ (and hence, also higher than his type). Proving that there is no mass of types who pool on a positive date requires rather involved arguments, and details are provided in Appendix A.

A type who discloses with delay chooses the latest time at which to disclose by balancing the marginal benefit from delay with the marginal cost. This is reflected in the equilibrium strategy of types below the cutoff θ. To understand this strategy consider a type $x<\theta$, who pretends to be a "slightly" lower type, $x-\varepsilon$. For each type $x-\varepsilon<y<x$ of the other individual, the net expected gain from this deviation is $((1-y)-x)f(y)$. The potential loss from this deviation is the wasteful delay that would occur if the other party's type is even lower than $x-\varepsilon$. At the limit when ε goes to zero, the ratio of the expected marginal gain to the expected marginal loss is $\frac{(1-2x)f(x)}{\delta xF(x)}$, which is the derivative of the equilibrium strategy of type x.

To prove that $\tau^*(x)$ is an equilibrium, we show that no type has an incentive to deviate, given that the other individual uses this strategy. The proof of uniqueness is more involved and details are provided in Appendix A.

5.2. Normative analysis

As evident from Proposition 6, the level of efficiency induced by a compromise rule depends entirely on the threshold. An important implication of this is that the partial ordering identified in Proposition 4 for the static game continues to predict the level of efficiency of equilibrium outcomes in the dynamic game as well.

Proposition 7. *Let r and r' be two regular compromise rules that satisfy (3), and let τ and τ' be the strategies in the symmetric BNE of the dynamic disclosure game associated with r and r' respectively. If $r'\succeq r$, then $\tau'(x)\le\tau(x)$, for each $x\in[0,1]$, and the equilibrium outcome associated with r' is Pareto superior.*

Corollary 1 thus extends to the dynamic game.

5.3. Dynamic vs. static

We start our comparison of the dynamic and static versions of the disclosure game by showing that, for any regular compromise rule satisfying condition (3), some types that are being disclosed in the static game are not getting disclosed right away in the dynamic game. In other words, the threshold derived for the static game is lower than the threshold derived for the dynamic game. Let θ_S and θ_D be the thresholds given by (1) and (4), respectively (i.e., the former is the cutoff of the one-shot simultaneous game, while the latter is the cutoff of the dynamic game).

Proposition 8. *If r is a regular compromise rule satisfying (3), then $\theta_S\le\theta_D$.*

Proposition 8 raises the following question: given a regular compromise rule satisfying (3), are individuals better off in the symmetric BNE of the static game or the dynamic game? The next proposition characterizes the players' ex-ante discounted payoffs both for the static and the dynamic disclosure game when f is uniform, as a function of the thresholds θ_S and θ_D respectively. Perhaps surprisingly, it follows that enforcing a hard deadline may be preferable, as is the case for instance for r_N, the most efficient regular rule.[16]

[16] It remains an open question to characterize the class of compromise rules for which enforcing a hard deadline is preferable, as well as understanding how welfare compares beyond the uniform distribution.

Proposition 9. *Suppose that the distribution of types is uniform. Let r be a regular compromise rule that satisfies (3). Each individual's ex-ante expected payoff is equal to* $\frac{1-\theta_S^2}{2}$ *in the static game, and equal to*

$$\frac{1}{2}[1-\theta_D^2 - 2e^{1/\theta_D}\cdot\theta_D^2\cdot E_i(-\frac{1}{\theta_D})] \tag{5}$$

in the dynamic game, where $E_i(x)$ *denotes the exponential integral. As a consequence, the outcome of the static game Pareto dominates that of the dynamic game for the most efficient regular rule,* $r = r_N$.

5.4. Dynamic disclosure with an opportunity to react

As a natural variant of our dynamic game, we study a situation where individuals have one last chance to disclose the option they are aware of right after the other has "spoken", i.e. right before r is implemented. Note that the strategies in this game are richer than those of the original dynamic game. As in the original game, they specify the latest period in which an individual would disclose if the other party has not done so. But in addition, for every history which ended with disclosure by the other party, an individual's strategy also specifies whether or not he would disclose as a function of the other party's disclosed type and the period of disclosure. To eliminate notational complications and unlikely off-equilibrium behavior, we focus on a slightly refined notion of BNE. Indeed, we will assume that type x discloses right after the other party has disclosed a type y if and only if $y > 1 - x$. In other words, we focus on equilibria in which an individual discloses immediately after the other party has disclosed whenever it is optimal for him to do so (whenever the payoff from the other party's option is lower than the payoff from his own). Given this restriction, strategies in a refined BNE are measurable functions $\tau : [0,1] \to \mathbb{R}_+ \cup \{\infty\}$, that describe when an individual discloses as a function of his type.

Proposition 10. *The modified dynamic disclosure game admits a unique refined symmetric Bayesian Nash equilibrium. The equilibrium disclosure strategy* t^* *for both individuals is the following:*

$$\mathrm{t}^*(x) = \begin{cases} 0 & \text{if } x \geq 1/2 \\ \int_x^{1/2} \frac{(1-2y)f(y)}{\delta y F(y)} dy & \text{if } x \leq 1/2. \end{cases}$$

By Proposition 10, the timing of disclosure in the unique refined symmetric BNE is *independent* of the compromise rule. As in the original dynamic game, types who delay have no incentive to pool. Hence, the rate of delay is independent of the compromise rule. The reason the threshold type is independent of the compromise rule is that the opportunity to respond gives types below $\frac{1}{2}$ an incentive to wait: if the other party disclosed an option that is less favorable, he would then respond by disclosing himself; otherwise, he would keep quiet. The rate at which types below $1/2$ disclose at equilibrium is the same as in the original dynamic disclosure game. The reason is that only types below $1/2$ disclose at a strictly positive time. Therefore such types do not use the opportunity to react, in which case incentives to reveal at t or at $t + \varepsilon$ are the same as those in Proposition 6. Establishing uniqueness is technically more involved and details are provided in the supplementary online appendix.

Notice that every type below $\frac{1}{2}$ delays the latest time at which he would disclose, relative to his timing of disclosure in the original dynamic game (where an individual cannot disclose immediately after his rival). Hence, for every compromise rule, the ex-ante expected payoff of an individual is lower in this dynamic game than in the original game discussed above. Again, one sees that more opportunities to speak can in fact be damaging in terms of welfare.

6. Beyond the benchmark

Knowing no or multiple options Consider a variant of our model where there is a fixed probability p that an individual is *not* aware of any option, while he is aware of an option as before with probability $1 - p$. Notice that the first individual's expected net gain of disclosing instead of withholding his type x (thus conditional on being aware of a collective decision) in the static disclosure game is equal to

$$ENG_1^p(x, \sigma_2) = px + (1-p)ENG_1(x, \sigma_2),$$

where ENG_1 was defined in Eq. (2). As explained after Proposition 1, our analysis builds on the fact that ENG_1 is increasing in x and decreasing in σ_2. Clearly, ENG_1^p inherits these properties from ENG_1. Hence, similar arguments imply that there is a unique symmetric BNE, which involves threshold strategies. The equilibrium threshold decreases as p increases (as it is easy to check that ENG_1^p is increasing in p), but is strictly positive for any $p < 1$. Comparisons between compromise rules in terms of the efficiency level they induce are unaffected by the introduction of the probability p. Similar arguments hold for the dynamic game as well. The expected net gain function of disclosing now instead of later retains similar properties independently of the value of p. The threshold above which types reveal right away decreases as p increases. The rate at which lower types disclose remains the same – $\frac{(1-2y)f(y)}{\delta y F(y)}$. Inefficiency in the form of delay persists for any $p < 1$, and comparisons between compromise rules in terms of the efficiency level they induce is unaffected by the introduction of the probability p.

Consider now a case where each individual knows about the feasibility of k feasible options whose associated payoffs fall on the line. Formally, i's type is a subset O_i of $[0,1]$ (each element of O_i determines i's own payoff, following the same convention as first described in Section 2) that contains k elements obtained from repeated independent draws that follow the density f. Suppose that the first individual has disclosed a subset X_1 of O_1, while the second individual has disclosed a subset X_2 of O_2. The feasible set in the space of utilities in that case is the smallest triangle that contains all the following vectors: $(0,0)$, $(x_1, 1-x_1)$, for all $x_1 \in X_1$, and $(1-x_2, x_2)$, for all $x_2 \in X_2$. A compromise rule is "welfarist" if final utilities depend only on the utility possibility set. Nash's (1950) bargaining model is welfarist, and so are in particular his, Raiffa's, and Kalai and Smorodinsky's solutions. An example of compromise rule that is not welfarist is one that picks the middle option when three have been disclosed. We restrict our discussion to the case of welfarist compromise rules. Hence the compromise depends only on the triangle with extreme points $(0,0)$, $(\bar{x}, 1-\bar{x})$ and $(1-\bar{y}, \bar{y})$, where $(\bar{x}, 1-\bar{x})$ is the utility pair that is most advantageous to the first individual among those that are available and $(1-\bar{y}, \bar{y})$ is the one that is most advantageous for the second individual.

Consider now the extended static or dynamic disclosure game where a pure strategy for either individual in the static game is any subset of his type, and a pure strategy for either individual in the dynamic game determines which subset of his type to disclose and at which time. It is not difficult to show that individuals' strategies depend only on the option that is most advantageous to them given those they are aware of. BNEs of the extended static or dynamic disclosure games are thus isomorphic to the BNEs of the original static or disclosure games where (single-valued) types are drawn according to the density f^k (which is the density of the maximum of k independent draws following f). This observation thus allows to extend all our results to cases where parties know multiple feasible options.

Beyond the straight line Recall that X represents the set of utility pairs associated with collective decisions that individuals may learn about.[17] Most of our results build on two key features of X. First it must be symmetric: $(y,x) \in X$ whenever $(x,y) \in X$. Second interests of the two individuals must be strictly opposed: if the first individual prefers x over x', then the second prefers x' over x. Any such set X can be described via a strictly decreasing isomorphism $g:[0,1] \to [0,1]$ such that $g(0)=1$, $g(1)=0$, and $g(1-x)=x$, for all $x \in [0,1]$. All our results and associated proofs, except Corollary 1, extend to this more general environment simply by replacing $1-y$ with $g^{-1}(y)$ and $1/2$ by u^*, where u^* is the unique fixed point of g (i.e. $u^* = g(u^*)$). For instance, Proposition 1 holds with the threshold

$$\theta = \sup\{x \in [0,u^*] \mid xF(x) + \int_{y=x}^{1} (r_1(x,y) - g^{-1}(y)) f(y) dy < 0\}.$$

The unique symmetric BNE in the dynamic disclosure game is given by

$$\tau^*(x) = \begin{cases} 0 & \text{if } x \geq \theta \\ \int_x^\theta \frac{(g^{-1}(y)-y)f(y)}{\delta y F(y)} dy & \text{if } x < \theta, \end{cases}$$

where

$$\theta = \sup\{x \in [0,u^*] | \int_{y=x}^{1} (r_1(x,y) - g^{-1}(y)) f(y) dy < 0\}.$$

Propositions 4 and 7 on the relative efficiency of different compromise rules hold with the partial ordering from Definition 2 (simply replace $1/2$ by u^* in that definition).

On the other hand, considering sets X different from a straight line has non-trivial implications as far as the optimal solution is concerned (see Corollary 1). First, the Nash solution need not be regular anymore. Second, even if it is regular, it may be dominated by other regular rules. Characterizing the most efficient regular compromise rule for any symmetric and decreasing g remains an open question.

More can be said if we further restrict g. In particular, we are able to characterize the most efficient regular compromise rule if we assume that g is differentiable and either convex or concave.

We start with the case where g is differentiable and *convex*. This case is interesting beyond mere generality. It allows to capture situations where an individual would not disclose an option which is only slightly worse for him but much better for the other individual (e.g., the first individual may disclose $(1,1)$ but not $(0,100)$), resulting in large efficiency losses ex-ante. In this case, r_{CF}, r_{KS} and r_N are all regular compromise rules (as shown in the supplementary online appendix). The next proposition identified the most efficient regular rule.

[17] To avoid any possible confusion, we emphasize that X is not the Pareto frontier of the set of feasible agreements as in a standard bargaining problem. Instead, the set of feasible (efficient) agreements consists of two points randomly drawn from X.

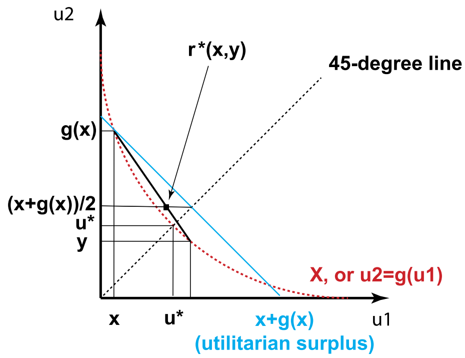

Fig. 1. The most efficient regular rule (r^*) when g is convex.

Let r^* be the compromise rule defined as follows. If two feasible options have been disclosed, it selects the lottery over these two options, which maximizes the expected payoff to the individual in the weaker position, subject to the constraint that the other individual's expected payoff is at least half the utilitarian surplus (see Fig. 1 above). This definition retains the idea of favoring the individual in the weaker position within the class of regular compromise rules. If only one option has been disclosed, r^* selects it with certainty. Observe that r^* coincides with the Nash solution when g is linear, but not in general.

Proposition 11. *r^* is the most efficient regular rule when g is convex.*

To analyze the case where g is differentiable and *concave*, we use the following "duality" argument. For any payoff pair $(u, g(u))$ we define a dual pair $(v, h(v))$ where $v \equiv 1 - u$ and $h(v) \equiv 1 - g(1 - v)$. It follows that $h(v)$ is differentiable, decreasing and convex. Let r be a regular compromise rule defined on the set of disclosable payoffs, $\{(v, h(v)) : v \in [0, 1]\}$. Define the "dual solution" to r as follows: for any pair of disclosed payoff pairs, $(u, g(u))$ and $(g^{-1}(u'), u')$,

$$d_i(u, u') = 1 - r_i(1 - u, 1 - u')$$

This mapping from the compromise rule r to its dual rule d preserves the regularity of the solutions as well as their ranking in terms of efficiency.

Proposition 12. *(i) If r is regular, then so is d, and (ii) for any pair of regular compromise rules, (r, r') and their dual rules (d, d'), we have that $r \succeq r'$ implies $d \succeq d'$.*

Let d^* be the dual of r^*, the regular compromise rule defined above, which is most efficient when g is convex. By Proposition 12, d^* is the most efficient regular compromise rule when g is concave. Note that the coin-flip rule and both Nash and Kalai–Smorodinsky, defined over a convex g, admit a dual regular compromise rule when g is concave. However, apart for the coin-flip rule, their dual does not correspond to the definition of the original rule (e.g., the dual of Nash does not select the payoff pair that maximizes the product of the individuals' payoffs).

Extending our analysis to cases where the option known to one individual may be Pareto inferior to the option known to the other individual would require developing entirely new arguments. Our analysis was indeed grounded in the fact that an individual's expected net gain increases with his type (see discussion following Proposition 1), while this property does not hold, for instance, when types are drawn from the uniform distribution on $[0, 1]^2$. Similarly, extending our analysis to non-symmetric distributions of types is left as an open problem for future research.

Appendix A

Lemma 1.

1. *$ENG_i(x, \sigma_{-i})$ is weakly increasing in x.*
2. *If the probability of disclosure is lower than 1 under σ_{-i}, then $ENG_i(x, \sigma_{-i})$ is strictly increasing in x.*
3. *If $\hat{\sigma}_{-i}(y) \geq \sigma_{-i}(y)$, for each $y \in [0, 1]$, then $ENG_i(x, \hat{\sigma}_{-i}) \leq ENG_i(x, \sigma_{-i})$.*

Proof. We assume $i = 1$. A similar argument applies to the second individual. The fact that it is non-decreasing in x follows immediately from the monotonicity condition on r. If $\{y \in X | \sigma_2(y) < 1\}$ has a strictly positive measure, then ENG is strictly increasing in x via the first term in its definition. The third property follows from the fact that $r_1(x, y) - (1 - y) \leq x$, for each $(x, y) \in [0, 1]^2$, which itself follows from the fact that $r_1(x, y) \leq \max\{x, 1 - y\}$, since r selects a convex combination between $(x, 1 - x)$ and $(1 - y, y)$. $\square$

For each $\epsilon \in [0,1]$, let σ_i^ϵ be the "threshold strategy" defined by $\sigma_i^\epsilon(x) = 0$ if $x < \epsilon$, and $\sigma_i^\epsilon(x) = 1$ if $x > \epsilon$. The next Lemma shows that there exists $\eta > 0$ small enough such that types below η prefer not to disclose when the other individual follows a threshold strategy whose threshold falls below η.

Lemma 2. *There exists $\eta > 0$ such that $ENG_i(x, \sigma_{-i}^\epsilon) < 0$, for all $x \in [0, \eta]$ and $\epsilon \in [0, \eta]$.*

Proof. Notice that $\sigma_{-i}(\epsilon) \geq \sigma_{-i}(\eta)$ for $\epsilon \leq \eta$, and hence $ENG_i(x, \sigma_{-i}^\epsilon) \leq ENG_i(x, \sigma_{-i}^\eta)$ by the third condition of Lemma 1. In turn, the right-hand side is lower or equal to $ENG_i(\eta, \sigma_{-i}^\eta)$ for $x \leq \eta$, by the first condition of Lemma 1. Hence we will be done after showing that $ENG_i(\eta, \sigma_{-i}^\eta) < 0$. We prove this inequality for $i = 1$. The case $i = 2$ follows from a symmetric argument. The first individual's expected net gain of disclosing when of type η is $\eta F(\eta) + \int_{y=\eta}^1 (r_1(\eta, y) - (1-y)) f(y)dy$. Suppose η is very small. The integral can be split into an integral for $y \in [\eta, 1-\eta]$, in which case the integrand is non-positive, and an integral for $y \in [1-\eta, 1]$, in which case the integrand is non-negative. The former term (for $y \in [\eta, 1-\eta]$) is smaller or equal to $\int_{y=\eta}^{1/2} (r_1(\eta, y) - (1-y)) f(y)dy$, which itself is smaller or equal to $\int_{y=\eta}^{1/2} (1/2 - (1-y)) f(y)dy$ since $r_1(\eta, y) \leq 1/2$, for all $y \in [\eta, 1/2]$ (by regularity). The latter term (for $y \in [1-\eta, 1]$) is smaller or equal to $\int_{y=1-\eta}^1 (\eta - (1-y)) f(y)dy$. To summarize, the first individual's expected net gain of disclosing when of a small type η is smaller or equal to $\int_{y=\eta}^{1/2} (1/2 - (1-y)) f(y)dy + \int_{y=1-\eta}^1 (\eta - (1-y)) f(y)dy$. Notice that this expression is continuous in x, and strictly negative at $\eta = 0$. Hence there must exist $\eta > 0$ small enough for which it remains negative. □

Proof of Proposition 1. (Existence) The first property from Lemma 1 implies that there exists a best response to any strategy, and that any such best response is a threshold strategy: if σ_i^* is a best response against σ_{-i}, then there exists a unique $\theta_i \in [0,1]$ such that $\sigma_i^*(x) = 0$, for each x such that $x < \theta_i$, and $\sigma_i^*(x) = 1$, for each $x \in [0,1]$ such that $x > \theta_i$. Such a threshold strategy will also be denoted $\sigma_i^{\theta_i}$. The existence of a symmetric BNE is thus equivalent to the existence of a fixed point to the correspondence that associates i's optimal threshold strategy to each of the opponent's threshold strategies, or $\theta_i = BR_i(\theta_{-i})$ for short. This will follow from Brouwer's fixed-point theorem after showing that BR_i is continuous. Let thus $(\theta(k))_{k \in \mathbb{N}}$ be a sequence of real numbers in $[0,1]$ that converges to some θ. Suppose on the other hand that $BR_i(\theta(k))$ converges to some $\theta' \neq BR_i(\theta)$. To fix ideas, we'll assume that $\theta' > BR_i(\theta)$ (a similar reasoning applies if the inequality is reversed). Hence there exists K such that $\frac{BR_i(\theta)+\theta'}{2} < BR_i(\theta(k))$, for all $k \geq K$, and $ENG_i(\frac{BR_i(\theta)+\theta'}{2}, \sigma^{\theta(k)}) < 0$. Taking the limit on k, we get

$$ENG_i(\frac{BR_i(\theta)+\theta'}{2}, \sigma^\theta) \leq 0,$$

by continuity of the integral with respect to its bounds, but which thus leads to a contradiction, since $\frac{BR_i(\theta)+\theta'}{2} > BR_i(\theta)$. Hence BR_i is indeed continuous, and admits a fixed-point.

(Uniqueness) Suppose, on the contrary, that one can find two symmetric BNE's. Let θ and θ' be the two corresponding common thresholds that the two individuals are using. Assume without loss of generality that $\theta' > \theta$, and let $\hat{\theta}$ be a number that falls between θ and θ'. Lemma 1 and the definition of the thresholds imply $0 < ENG_1(\hat{\theta}, \sigma_2^\theta) \leq ENG_1(\hat{\theta}, \sigma_2^{\theta'}) < 0$, which is impossible. This establishes the uniqueness of the symmetric BNE.

(Threshold in $]0, 1/2]$) We know from Lemma 2 that the equilibrium threshold must be strictly positive. We now establish that it is no greater than 1/2. First observe that $\int_{y=1/2}^1 [r_1(\frac{1}{2}, y) - (1-y)] f(y)dy \geq 0$, because $r_1(\frac{1}{2}, y) \geq 1-y$, for all $y \geq 1/2$. Adding $1/2F(1/2)$ to this expression leads to a strictly positive number, and hence $BR_1(1/2) \leq 1/2$. We also know from Lemma 2 that $BR_1(\eta) \geq \eta$. Hence BR_i's fixed point must belong to $[\eta, 1/2]$.

(Closed-Form Formula) We know that the threshold θ is given by $\theta = BR_1(\theta) = \sup\{x \in [0, 1/2] \mid ENG_1(x, \sigma^\theta) < 0\}$. Lemma 1 implies that $ENG_1(x, \sigma^x) \leq ENG_1(x, \sigma^\theta)$, for all $x < \theta$, and $ENG_1(x, \sigma^\theta) \leq ENG_1(x, \sigma^x)$, for all $\theta < x$. Hence $\theta = \sup\{x \in [0, 1/2] \mid ENG_1(x, \sigma^x) < 0\}$, which establishes Eq. (1). □

Proof of Proposition 2. Let Σ be the set of strategies, for either individual,[18] that survive the iterated elimination of strictly dominated strategies. Let then $\theta = \sup\{x \in [0,1] | (\forall \sigma \in \Sigma) : \sigma = 0$ almost surely on $[0,x]\}$ and $\theta' = \inf\{x \in [0,1] | (\forall \sigma \in \Sigma) : \sigma = 1$ almost surely on $[x,1]\}$. Obviously, $\theta \leq \theta'$. Observe also that $\theta \leq BR_i(BR_i(\theta))$ if the disclosure game admits a unique BNE. Otherwise, the function that associates $x - BR_i(BR_i(x))$ to each x between 0 and θ is strictly positive at θ and non-positive at 0, and hence admits a zero by the intermediate value theorem. Let thus θ^* be an element of $[0, \theta)$ such that $\theta^* = BR_i(BR_i(\theta^*))$. Notice that the pair of strategies $(\sigma^{\theta^*}, \sigma^{BR_2(\theta^*)})$ then forms a BNE, which implies that $\sigma^{\theta^*} \in \Sigma$ and contradicts the definition of θ.

[18] Indeed, the set of strategies that survive the iterated elimination of strictly dominated strategies is the same for both individuals because the game is symmetric.

Any strategy in Σ for i's opponent has him withhold his information for almost every type between 0 and θ. The more his opponent reveals, the lower i's expected net gain, according to Lemma 1. Hence if individual i wants to disclose his type when his opponent uses σ^θ, then a fortiori he wants to disclose it when his opponent plays some strategy in Σ (because there is more disclosure with σ^θ than with any strategy from Σ). This means that against any strategy in Σ, individual i's best response satisfies that he discloses his type whenever it is above $BR_i(\theta)$. Hence $\theta' \le BR_i(\theta)$.

The third property in Lemma 1 implies that BR_i is non-increasing, and hence $BR_i(\theta') \ge BR_i(BR_i(\theta))$. In the same way we proved that $\theta' \le BR_i(\theta)$, Lemma 1 and the definition of θ implies that $\theta \ge BR_i(\theta')$, and hence $\theta \ge BR_i(BR_i(\theta))$, by transitivity. Combining this with our earlier observation, we conclude that $\theta = BR_i(BR_i(\theta))$ and hence the pair of strategies $(\sigma^\theta, \sigma^{BR_2(\theta)})$ forms a BNE. Uniqueness of the BNE implies that this is in fact the symmetric BNE. Hence we must also have that $\theta = BR_i(\theta)$, which implies that $\theta' = \theta$, and we are done proving that the unique symmetric BNE is also the unique profile of strategies that survive the iterated elimination of strictly dominated strategies when the disclosure game admits a unique BNE. □

Proof of Proposition 3. Arguments developed in the proof of Proposition 1 imply that any BNE must involve threshold strategies. Hence, a BNE is efficient only if at least one of the two individuals follows a fully revealing strategy. To fix ideas, suppose that we have a BNE in which the second individual systematically reveals the option he is aware of. Note that the first individual's expected net gain from disclosing when of type $\frac{1}{2}$ is given by

$$\int_{y=0}^{1} [r_1(\frac{1}{2}, y) - (1-y)] f(y) dy.$$

This expression can be decomposed into two components: one where the opponent's type is below $\frac{1}{2}$, and another where his type is above $\frac{1}{2}$. The second component may be rewritten as follows. First, by anonymity, $r_1(\frac{1}{2}, y) = r_2(y, \frac{1}{2})$. Second, by ex-post efficiency, $r_2(y, \frac{1}{2}) = 1 - r_1(y, \frac{1}{2})$. Third, by symmetry of r, $r_1(y, \frac{1}{2}) = r_1(\frac{1}{2}, 1-y)$. Making the change of variable $y' \equiv 1 - y$, it follows that the net expected gain of type $\frac{1}{2}$ equals

$$\int_{y=0}^{\frac{1}{2}} [r_1(\frac{1}{2}, y) - (1-y)] f(y) dy + \int_{y'=0}^{\frac{1}{2}} [1 - r_1(\frac{1}{2}, y') - y'] f(1-y') dy. \qquad (6)$$

This expression is strictly positive, since $f(x) > f(1-x)$, for all $x \ge 1/2$, and hence the threshold for the first individual's best response strategy is strictly smaller than $1/2$. Let's call it θ. Then the second individual's expected net gain of revealing when of type $y = 0$ is equal to

$$\int_{x=\theta}^{1} (r_2(x, 0) - (1-x)) f(x) dx.$$

Observe that $\int_{x=1/2}^{1} (r_2(x,0) - (1-x)) f(x) dx \le 0$, since $r_2(x, 0) \le (1-x)$, for all x. Also, it must be that $\int_{x=\theta}^{1/2} (r_2(x,0) - (1-x)) f(x) dx < 0$ for any regular r, as $r_2(x, 0) \le r_2(0, 0) = 1/2 < 1 - x$, for all $0 < x < 1/2$. Hence systematic revelation cannot be a best response when the other individual uses a threshold strategy that discloses some types below $1/2$, and there is no way to achieve efficiency in any BNE of the disclosure game. □

Proof of Proposition 4. Recall from Proposition 1 that the unique symmetric BNE of the disclosure game associated with any regular compromise rule involves threshold strategies, whose common threshold falls in the interior of $[0, \frac{1}{2}]$. Let θ be the threshold associated with r, and θ' be the threshold associated with r'. Notice that

$$ENG_1^{r}(\theta', \sigma_2^{\theta'}) \le 0. \qquad (7)$$

Indeed, this inequality actually holds pointwise, since $r' \succeq r$ and the second individual withholds his information when $y < \theta'$, and is thus preserved through summation.

We now conclude the proof by showing that $\theta \ge \theta'$. Suppose, on the contrary, that $\theta < \theta'$, and let $\hat{\theta}$ be a number that falls between θ and θ'. Remember from Lemma 1 that an individual's expected net gain is increasing in his own type. Inequality (7) thus implies that $ENG_1^{r}(\hat{\theta}, \sigma_2^{\theta'}) < 0$. Remember also from Lemma 1 that an individual's expected net gain does not increase when the opponent reveals more, and hence $ENG_1^{r}(\hat{\theta}, \sigma_2^{\theta}) < 0$. This contradicts the fact that the threshold strategies associated with θ form a BNE of the disclosure game associated with r (as it should be optimal for the first individual to reveal his type $\hat{\theta}$ since it is larger than θ). □

Proof of Corollary 1. Let x be a number smaller or equal to 1/2. We must prove that $r_N(x, y)$ is more advantageous to the first individual than $r(x, y)$, for all $y \geq x$. This is obvious when $y \geq 1/2$ since the Nash solution picks the right-most option in that region. Since r is anonymous, it must be that $r(x, x) = (1/2, 1/2)$. Monotonicity implies that $r_1(x, y) \leq 1/2$ for each $y \in [x, 1/2]$, hence, the desired inequality when compared to the Nash solution which always picks 1/2 in that region. □

Proof of Proposition 5. If f is symmetric, then expression (6) is equal to zero, and hence the first individual's best response to the fully revealing strategy is to reveal if and only if his type is larger or equal to 1/2. Let's check now that systematic revelation is a best response for the second individual. Notice that

$$\int_{x=1/2}^{1} (r_2(x,0) - (1-x)) f(x) dx$$

cannot be strictly negative, as the second individual's expected net gain of disclosing would then be negative when his type is very small. Given that $r_2(x, 0) \leq 1 - x$, for all $x \in [1/2, 1]$ and that r_2 is continuous, it must thus be that $r_2(x, 0) = 1 - x$, for all $x \in [1/2, 1]$. The third regularity condition implies that $r_2(x, y) = 1 - x$, for all $y \leq 1/2$ and all $x \in [1/2, 1 - y]$. The second regularity condition then implies that $r_2(x, y) = \max\{1 - x, y\}$ if $\max\{1 - x, y\} \leq 1/2$ and $= \min\{1 - x, y\}$ if $\min\{1 - x, y\} \geq 1/2$. Similar conditions apply for the first individual's payoffs, by anonymity. Consider now a case where both x and y are no larger than 1/2. The third regularity condition implies that the first individual's payoff is no larger than $r_1(1/2, y) = 1/2$ and the second individual's payoff is no larger than $r_2(x, 1/2) = 1/2$. Hence $r(x, y) = (1/2, 1/2)$. Anonymity implies that a similar argument applies when both x and y are no smaller than 1/2. Hence r must coincide with the Nash solution. When r is the Nash solution, the second individual is indifferent between revealing or not when of type $y = 0$ given that the opponent systematically reveals. Hence full disclosure is a best response and we have identified an efficient BNE. □

Proof of Proposition 6 (Existence). We prove that the strategy τ^* is indeed part of a symmetric BNE. We start by showing that reporting at $\tau^*(x)$ is optimal, for any $x \in [0, \theta[$. Consider first the possibility of revealing at positive times. The function τ^* being invertible on $[0, \theta[$, we can identify any positive time with the type speaking at that time. The expected utility from revealing at $\tau^*(z)$ when of type x is equal to

$$U(z|x) := xF(z)e^{-\delta\tau^*(z)} + \int_{y=z}^{1} (1-y)e^{-\delta\tau^*(y)} f(y) dy,$$

for each $z \in [0, \theta[$. This expression is differentiable, and the derivative is equal to

$$xf(z)e^{-\delta\tau^*(z)} - \delta x(\tau^*)'(z)F(z)e^{-\delta\tau^*(z)} - (1-z)f(z)e^{-\delta\tau^*(z)},$$

or

$$\frac{(1-z)}{z} f(z)(x-z)e^{-\delta\tau^*(z)}$$

after rearranging the terms and using the definition of τ^* to compute $(\tau^*)'$. We see that the first order condition is satisfied at $z = x$, and that the derivative is positive when $z < x$ and negative when $x < z$. Hence there is no profitable deviation to a positive time different from $\tau^*(x)$, when of type x. Deviating to report at zero is not profitable either, as the expected payoff in that case is $xF(\theta) + \int_{y=\theta}^{1} r_1(x, y) f(y) dy$, which is equal to $U(\theta|x) + \int_{y=\theta}^{1} (r_1(x, y) - (1 - y)) f(y) dy$. For any $\epsilon > 0$ small enough, using the third regularity condition, this last expression is lower or equal to

$$U(\theta|x) + \int_{y=\theta-\epsilon}^{1} (r_1(\theta-\epsilon, y) - (1-y)) f(y) dy + \int_{y=\theta-\epsilon}^{\theta} ((1-y) - r_1(\theta-\epsilon, y)) f(y) dy.$$

The second term is negative, for all $\epsilon > 0$, by definition of θ. Hence, taking the limit when ϵ decreases to zero, we get that the expected utility of reporting at zero is no greater than $U(\theta|x)$, which in turn, by our previous reasoning, is smaller than the expected utility of reporting at $\tau(x)$. This establishes the optimality of τ^*, for any type strictly in between 0 and θ.

Consider now a type $x \in]\theta, 1]$. The expected utility of revealing at a time t is equal to $U(z|x)$, where z is the unique real number in $[0, \theta[$ such that $\tau^*(z) = t$. Our earlier reasoning regarding U's derivative implies that this expected utility is strictly lower than $U(\theta|x)$ (since $z < \theta \leq x$), which is equal to $xF(\theta) + \int_{y=\theta}^{1} (1 - y) f(y) dy$. Notice that $\int_{y=\theta}^{\theta+\epsilon} (r_1(x, y) - (1 - y)) f(y) dy$ converges to zero as ϵ decreases to zero. Hence $U(z|x) < U(\theta|x) + \int_{y=\theta}^{\theta+\epsilon} (r_1(x, y) - (1 - y)) f(y) dy$, for any $\epsilon > 0$ that is small enough. This in turn implies that

$$U(z|x) < U(\theta|x) + \int_{y=\theta}^{\theta+\epsilon} (r_1(x, y) - (1 - y)) f(y)dy + \int_{y=\theta+\epsilon}^{1} (r_1(\theta+\epsilon, y) - (1 - y)) f(y)dy,$$

by definition of θ. Applying now the third regularity condition (ϵ is small enough so that $x > \theta + \epsilon$), we conclude that

$$U(z|x) < U(\theta|x) + \int_{y=\theta}^{\theta+\epsilon} (r_1(x, y) - (1 - y)) f(y)dy + \int_{y=\theta+\epsilon}^{1} (r_1(x, y) - (1 - y)) f(y)dy.$$

The right-hand side is the expected utility for type x of revealing at zero, and we have thus proved the optimality of τ^* for any type no smaller than θ.

Finally, if $x = \theta$, then a similar reasoning as in the last paragraph implies that revealing at a positive time leads to a payoff that is no larger than $U(\theta|\theta) = \theta F(\theta) + \int_{y=\theta}^{1}(1 - y) f(y)dy$. Revealing at zero gives the expected payoff $\theta F(\theta) + \int_{y=\theta}^{1} r_1(x, y) f(y)dy$. The condition $\int_{y=\theta}^{1}(r_1(\theta, y) - (1 - y)) f(y)dy \geq 0$ thus guarantees that revealing at zero is optimal for type θ. □

Proof of Proposition 6 (Uniqueness). Let r be a regular compromise rule that satisfies condition (3), and let τ be a strategy that is part of a symmetric BNE in the original dynamic game. We have to show that $\tau = \tau^*$. We proceed in various steps.

Step 1. *The probability of having a type disclose at any given strictly positive time or at infinity (i.e. not disclose at all) is zero. Formally, $\int_{y\in\tau^{-1}(\infty)} f(y)dy = 0$ and $\int_{y\in\tau^{-1}(t)} f(y)dy = 0$, for all $t > 0$.*

Proof. We start by proving that $\int_{y\in\tau^{-1}(\infty)} f(y)dy = 0$. Suppose, to the contrary, that $\int_{y\in\tau^{-1}(\infty)} f(y)dy > 0$. Let $x > 0$ be such that $\tau(x) = \infty$. The first individual's expected net gain from disclosing at a time t instead of ∞ is:

$$e^{-\delta t} x \int_{y\in\tau^{-1}(\infty)} f(y)dy + \int_{y\in\tau^{-1}(]t,\infty[)} (e^{-\delta t}x - e^{-\delta\tau(y)}(1 - y)) f(y)dy$$
$$+ \int_{y\in\tau^{-1}(\{t\})} e^{-\delta t}(r_1(x, y) - (1 - y)) f(y)dy,$$

which is equal to $e^{-\delta t}$ times

$$x \int_{y\in\tau^{-1}(\infty)} f(y)dy + \int_{y\in\tau^{-1}(]t,\infty[)} (x - e^{-\delta(\tau(y)-t)}(1 - y)) f(y)dy$$
$$+ \int_{y\in\tau^{-1}(\{t\})} (r_1(x, y) - (1 - y)) f(y)dy,$$

which is greater or equal to

$$x \int_{y\in\tau^{-1}(\infty)} f(y)dy - \int_{y\in\tau^{-1}(]t,\infty[)} f(y)dy,$$

since both x and $r_1(x, y)$ are non-negative, and both $1 - y$ and $e^{-\delta(\tau(y)-t)}(1 - y)$ are no larger than 1. The first term of this last expression is strictly positive, and independent of t, while the second can be made as small as needed by taking t large enough, as $\lim_{t\to\infty} \int_{y\in\tau^{-1}(]t,\infty[)} f(y)dy = 0$ by measurability of τ.

We now check the second part of this first step: if $t \in]0, \infty[$, then $\int_{y\in\tau^{-1}(t)} f(y)dy = 0$. Let $\bar{x}$ be the supremum of $\tau^{-1}(t)$, and $\underline{x}$ be the infimum of $\tau^{-1}(t)$. For expositional convenience, we start by assuming that both the infimum and the supremum are reached in $\tau^{-1}(t)$, but we will show at the end of the proof how our argument extends to the more general case.

We start by assuming that $\underline{x} \leq 1 - \bar{x}$. Hence $1 - y \geq r_1(\underline{x}, y)$, for all $y \in \tau^{-1}(t)$. In addition, $1 - y > r_1(\underline{x}, y)$ for each $y \in \tau^{-1}(t)$ such that $y < 1/2$, as a consequence of the third regularity condition (Monotonicity), and the fact that $r_1(\underline{x}, \underline{x}) = 1/2$. We now prove that $\int_{y\in\tau^{-1}(t)\cap[0,1/2[} f(y)dy = 0$. Otherwise, the previous reasoning implies that $\int_{y\in\tau^{-1}(t)}((1 - y) - r_1(\underline{x}, y)) f(y)dy > 0$. Given that τ is a measurable function, we know that

$$\lim_{k\to\infty} \int_{y\in[0,1] \text{ s.t. } t<\tau(y)\leq t+\frac{1}{k}} f(y)dy = \int_{y\in[0,1] \text{ s.t. } t<\tau(y)\leq \lim_{k\to\infty} t+\frac{1}{k}} f(y)dy = 0,$$

and hence one can always find a k as large as necessary such that there is a very small probability for the other individual to speak in between t and $t+\frac{1}{k}$. The first individual's expected net gain of disclosing at $t+\frac{1}{k}$ instead of t when of type $\underline{x}$ is

$$\underline{x}(e^{-\delta(t+\frac{1}{k})} - e^{-\delta t}) \int_{y\in\tau^{-1}(]t+\frac{1}{k},\infty])} f(y)dy + \int_{y\in\tau^{-1}(t+\frac{1}{k})} (e^{-\delta(t+\frac{1}{k})} r_1(\underline{x}, y) - e^{-\delta t}\underline{x}) f(y)dy$$
$$+ \int_{y\in\tau^{-1}(]t,t+\frac{1}{k}[)} (e^{-\delta\tau(y)}(1-y) - e^{-\delta t}\underline{x}) f(y)dy + \int_{y\in\tau^{-1}(t)} e^{-\delta t}((1-y) - r_1(\underline{x}, y)) f(y)dy,$$

which is larger or equal to $e^{-\delta t}$ times

$$\underline{x}(e^{-\delta/k} - 1) \int_{y\in\tau^{-1}(]t+\frac{1}{k},\infty])} f(y)dy - \underline{x} \int_{y\in\tau^{-1}(]t,t+\frac{1}{k}])} f(y)dy$$
$$+ \int_{y\in\tau^{-1}(t)} ((1-y) - r_1(\underline{x}, y)) f(y)dy,$$

as it is indeed easy to check that the integrand of the second and third terms from the previous expression are both larger or equal to $-\underline{x}e^{-\delta t}$. The first two terms of the last expression can be made as small as needed by choosing a k large enough, while the third one is strictly positive independently of k, and hence the possibility of a profitable deviation, which contradicts the fact that τ is part of a symmetric BNE. Hence we have proved, by contradiction, that $\int_{y\in\tau^{-1}(t)\cap[0,1/2[} f(y)dy = 0$, and hence that $\int_{y\in\tau^{-1}(t)} f(y)dy = \int_{y\in\tau^{-1}(t)\cap[1/2,1]} f(y)dy$. If $\bar{x} \leq 1/2$, then we are done proving that $\int_{y\in\tau^{-1}(t)} f(y)dy = 0$. Let's thus assume that $\bar{x} > 1/2$.

Notice that $\bar{x} \geq r_1(\bar{x}, y)$, for each $y \in \tau^{-1}(t)$ such that $y \geq 1/2$. In fact, $\bar{x} > r_1(\bar{x}, y)$ for each $y \in \tau^{-1}(t)$ such that $y > 1/2$, as a consequence of condition (4) in the main paper, and the third regularity condition (Monotonicity). Hence $\int_{y\in\tau^{-1}(t)}(\bar{x} - r_1(\bar{x}, y)) f(y)dy > 0$ if $\int_{y\in\tau^{-1}(t)\cap]1/2,1]} f(y)dy > 0$. In that case, one can construct a profitable deviation to a $t' < t$ for type $\bar{x}$ (similar argument to the one developed in the previous paragraph). To avoid this contradiction, one must accept that $\int_{y\in\tau^{-1}(t)\cap]1/2,1]} f(y)dy = 0$. Combined with the result of the previous paragraph, one concludes that $\int_{y\in\tau^{-1}(t)} f(y)dy = 0$, as desired.

A similar argument applies in the case where $\underline{x} \geq 1 - \bar{x}$, except that one must start to work with $\bar{x}$ to show that $\int_{y\in\tau^{-1}(t)\cap]1/2,1]} f(y)dy = 0$, and then work with $\underline{x}$ to conclude.

We now consider the case where $\underline{x}$ and $\bar{x}$ do not necessarily belong to $\tau^{-1}(t)$. Again, we provide the argument only for the case were $\underline{x} \leq 1 - \bar{x}$, a similar argument applying if the inequality is reversed. Let $(\underline{x}_n)_{n\in\mathbb{N}}$ be a decreasing sequence in $\tau^{-1}(t)$ that converges to $\underline{x}$, and let $(\bar{x}_n)_{n\in\mathbb{N}}$ be an increasing sequence in $\tau^{-1}(t)$ that converges to $\bar{x}$ such that $\underline{x}_n \leq 1 - \bar{x}_n$, for each n. For notational simplicity, let α_n be the following real number:

$$\alpha_n := \int_{y\in\tau^{-1}(t)\cap[\underline{x}_n,\bar{x}_n]} ((1-y) - r_1(\underline{x}_n, y)) f(y)dy,$$

for each $n \in \mathbb{N}$. Notice first that these numbers are non-decreasing in n. Indeed, consider $m < n$. We have:

$$\alpha_n = \int_{y\in\tau^{-1}(t)\cap[\underline{x}_n,\underline{x}_m]} ((1-y) - r_1(\underline{x}_n, y)) f(y)dy + \int_{y\in\tau^{-1}(t)\cap[\underline{x}_m,\bar{x}_m]} ((1-y) - r_1(\underline{x}_n, y)) f(y)dy$$
$$+ \int_{y\in\tau^{-1}(t)\cap[\bar{x}_m,\bar{x}_n]} ((1-y) - r_1(\underline{x}_n, y)) f(y)dy.$$

Since $\underline{x}_n \leq 1 - \bar{x}_n$, we must have $r_1(\underline{x}_n, y) \leq 1 - y$, for each $y \in [\underline{x}_n, \bar{x}_n]$, and hence the first and the third terms must be non-negative. The third regularity condition also implies that the second term is larger or equal to α_m, since $\underline{x}_m \geq \underline{x}_n$, and hence $\alpha_n \geq \alpha_m$, as desired.

We now show that $\int_{y\in\tau^{-1}(t)\cap[0,1/2]} f(y)dy = 0$. Otherwise, there exists N such that $\int_{y\in\tau^{-1}(t)\cap[0,1/2]\cap[\underline{x}_n,\bar{x}_n]} f(y)dy > 0$, for each $n \geq N$. The reasoning that we did at the beginning of the proof when the infimum and the supremum are reached implies that $\alpha_n > 0$, for each $n \geq N$, and in particular $\alpha_N > 0$. Notice that

$$\int_{y\in\tau^{-1}(t)} ((1-y)-r_1(\underline{x}_n,y))f(y)dy = \alpha_n + \int_{y\in\tau^{-1}(t)\setminus[\underline{x}_n,\bar{x}_n]} ((1-y)-r_1(\underline{x}_n,y))f(y)dy,$$

for each $n \geq N$. The first term is larger or equal to α_N, which is strictly larger than 0 and independent of n, while the second term converges towards zero as n increases, since the integrand is bounded and $\int_{y\in\tau^{-1}(t)\setminus[\underline{x}_n,\bar{x}_n]} f(y)dy$ converges towards zero, and we are done proving that the expression on the left-hand side must be strictly positive for n large enough. As before, this implies that the first individual of type $\underline{x}_n$ prefers to disclose his type slightly later than at t, thereby contradicting the definition of a BNE. It must thus be the case that $\int_{y\in\tau^{-1}(t)\cap[0,1/2]} f(y)dy = 0$, as desired.

Adapting the argument to show that $\int_{y\in\tau^{-1}(t)\cap[1/2,1]} f(y)dy = 0$ when the infimum and the supremum are not reached, and thereby conclude the proof, is similar and left to the reader. □

Step 2. *τ is strictly decreasing with respect to time: if $x' > x$ and $\tau(x) > 0$, then $\tau(x') < \tau(x)$; if $x' > x$ and $\tau(x) = 0$, then $\tau(x') = 0$.*

Proof. Let $x, x' \in [0,1]$ such that $x' > x$ and $\int_{y\in[0,1] \text{ s.t. } \tau(y)>\tau(x)} f(y)dy > 0$.

By definition of BNE, $ENG_1(\tau(x') \text{ vs. } \tau(x), x') \geq 0$. If $\tau(x') > \tau(x)$, then[19] $ENG_1(\tau(x') \text{ vs. } \tau(x), x) > 0$, which contradicts the optimality of reporting at $\tau(x)$ when of type x. Hence, it must be that $\tau(x') \leq \tau(x)$.

Suppose now that $\tau(x) > 0$. We know from the previous paragraph that $\tau(x'') \leq \tau(x)$, for all $x'' \in]x, x'[$. Step 1 implies that there exists $x'' \in]x, x'[$ such that $\tau(x'') < \tau(x)$. The reasoning from the previous paragraph implies that $\tau(x') \leq \tau(x'')$, and hence $\tau(x') < \tau(x)$.

We have thus established the two desired properties, but under the assumption that $\int_{y\in[0,1] \text{ s.t. } \tau(y)>\tau(x)} f(y)dy > 0$. We now show that this inequality must in fact hold for any $x > 0$. Suppose first that x is such that $\tau(x) = 0$. If the inequality does not hold, then it means that the opponent will reveal his type with probability 1 at time 0. Then it is easy to check that $\int_{y\in[0,1]} r_1(x,y)f(y)dy < \int_{y\in[0,1]}(1-y)f(y)dy$, for any $x \in [0,1]$ that is small enough. A reasoning similar to the one developed in the third paragraph of the proof of Step 1 would imply a contradiction, namely that a slight delay is a profitable deviation for any such x. Consider now an x such that $\tau(x) > 0$, let $t^* = \inf_{y\in[0,x]} \tau(y)$, and let $(x_k)_{k\in\mathbb{N}}$ be a sequence in $[0,x]$ such that $(\tau(x_k))_{k\in\mathbb{N}}$ decreases towards t^* as k tends to infinity. Since τ is measurable, we have:

$$\lim_{k\to\infty} \int_{y\in\tau^{-1}(]\tau(x_k),\infty])} f(y)dy = \int_{y\in\tau^{-1}(]\lim_{k\to\infty}\tau(x_k),\infty])} f(y)dy = \int_{y\in\tau^{-1}(]t^*,\infty])} f(y)dy.$$

Notice that the right-most expression must be strictly positive. We just proved this if $t^* = 0$, while, if $t^* > 0$, then the opponent does not speak before t^* if his type is no greater than x, and the probability of him speaking at t^* is zero, by Step 1. Hence there exists $K \in \mathbb{N}$ such that $\int_{y\in[0,1] \text{ s.t. } \tau(y)>\tau(x_k)} f(y)dy > 0$, for all $k \geq K$. The result from the previous paragraph implies that $\tau(x) \leq \tau(x_k)$, for all such k's, and hence $\tau(x) = t^*$, and $\int_{y\in[0,1] \text{ s.t. } \tau(y)>\tau(x)} f(y)dy > 0$. □

Step 3. *Let $\alpha = \inf\{x \in [0,1] | \tau(x) = 0\}$. Then τ is continuous on $]0,\alpha[$, and $\lim_{x\to\alpha_-} \tau(x) = 0$.*

Proof. Let $x \in]0,\alpha]$, and let $(x_k)_{k\in\mathbb{N}}$ be a sequence in $[0,x]$ that converges to x. Step 2 implies that $\tau(x_k) \geq \tau(x)$, for all $k \in \mathbb{N}$. Suppose, to the contrary of what we want to prove, that there exists $\eta > 0$ and $K \in \mathbb{N}$ such that $\tau(x_k) > \tau(x) + \eta$, for all $k \geq K$. This implies that no type reveals after $\tau(x)$ and before $\tau(x) + \eta$. Indeed, suppose on the contrary that there exists y such that $\tau(y) \in]\tau(x), \tau(x) + \eta[$. Step 2 implies that y is strictly smaller than x, and hence there exists $k \geq K$ such that $y < x_k < x$. Step 2 implies that $\tau(x_k) < \tau(y) < \tau(x) + \eta$, which contradicts the definition of K. Consider now a type y for which $\tau(y)$ is very close to the $\inf\{\tau(z) | \tau(z) \geq \tau(x) + \eta\}$ (i.e. y is smaller than x, but very close to it). Then revealing a bit earlier, let's say at $\tau(x) + \frac{\eta}{2}$ instead of $\tau(y)$, is a profitable deviation since the loss, coming from the opponent's types between y and x, can be made as small as needed, while the gain is larger than the gain from getting y earlier by at least $\eta/2$ units of time for all the opponent's type who reveal after $\tau(y)$ (y is strictly positive if close enough to x, and so there is a positive probability that the opponent reveals after $\tau(y)$). This contradicts the optimality of revealing y at $\tau(y)$, and hence we have established left-continuity on $]0,\alpha[$, and that $\lim_{x\to\alpha_-} \tau(x) = 0$. A similar reasoning applies to show the right-continuity on $]0,\alpha[$. □

Step 4. *$\tau(x) = 0$ if and only if $x \in [\theta, 1]$, where*

$$\theta = \sup\{x \in [0,1/2] | \int_{y=x}^{1} (r_1(x,y) - (1-y))f(y)dy < 0\}.$$

[19] The second term in the definition of the expected net gain, as stated before the statement of this proposition, is zero, by Step 1.

Proof. Observe first that the function $g : [0, 1/2] \to \mathbb{R}$ that associates $\int_x^1 (r_1(x, y) - (1 - y)) f(y) dy$, to any $x \in [0, 1/2]$, is strictly increasing. Suppose that $x' > x$. We have:

$$g(x') = \int_{y=x'}^{1} (r_1(x', y) - (1 - y)) f(y) dy \geq \int_{y=x'}^{1} (r_1(x, y) - (1 - y)) f(y) dy$$
$$> \int_{y=x}^{1} (r_1(x, y) - (1 - y)) f(y) dy = g(x).$$

The weak inequality follows from the third regularity condition, while the strict inequality follows from the fact that $r_1(x, y) - (1 - y) < 0$, for each $y \in]x, x'[$, as $1 - y > 1/2$ and $r_1(x, y) \leq 1/2$ (as a consequence of the second and third regularity conditions), for all such y's. Notice also that $g(0) < 0$. Indeed, $r_1(0, y) \leq 1 - y$, for all $y \in [1/2, 1]$, by the first regularity condition, and $r_1(0, y) \leq 1/2 < 1 - y$, for all $y \in [0, 1/2[$, by the second and third regularity conditions. Notice finally that $g(1/2) \geq 0$, as $r_1(1/2, y) \geq 1 - y$, for each $y \in [1/2, 1]$, by the first regularity condition. Hence θ is well-defined, $g(x) < 0$, for each $x \in [0, 1/2]$ such that $x < \theta$, and $g(x) > 0$, for each $x \in [0, 1/2]$ such that $x > \theta$.

We now prove that $\tau(x) > 0$, for each $x < \theta$. Otherwise, there exists $x < \theta$ such that $\tau(x) = 0$. Then $g(x) < 0$, and hence

$$\int_{y=\alpha}^{1} r_1(x, y) < \int_{y=\alpha}^{1} (1 - y) f(y) dy,$$

where $\alpha = \inf\{y \in [0, 1] | \tau(y) = 0\}$, because $r_1(x, y) \leq 1 - y$, for each $y \in [\alpha, x]$, by the first regularity condition. A reasoning similar to the one we did in the third paragraph in the proof of Step 1 implies that an individual of type x can improve his payoff by reporting at some small positive time rather than at zero, thereby contradicting the optimality of τ. Hence $\tau(x) > 0$, for each $x < \theta$, as desired.

We now prove that $\tau(x) = 0$, for each $x > \theta$. First notice that $\tau(x) = 0$, for each $x > 1/2$. Suppose, on the contrary, that $\tau(x) > 0$, for some $x > 1/2$. The expected net gain of reporting at 0 instead is strictly positive, as $r_1(x, y) - (1 - y) \geq 0$, for all the opponent's types y that report at 0, and $x > 1 - y$, for all the opponent's types $y > x$ that report at a positive time lower than $\tau(x)$. So $\tau(x) = 0$, for each $x > 1/2$, and we have proved the statement for $\theta = 1/2$. Suppose now that $\theta < 1/2$. As before, let $\alpha = \inf\{y \in [0, 1] | \tau(y) = 0\}$. We know that $\alpha \leq 1/2$. Suppose, to the contrary of what we want to prove, that $\alpha > \theta$. Let then x be smaller than α, but very close to it. Hence $\tau(x) > 0$. The expected net gain of revealing at zero instead is equal to:

$$\int_{y=\alpha}^{1} (r_1(x, y) - (1 - y)) f(y) dy + \int_{y=x}^{\alpha} (x - e^{-\delta\tau(y)}(1 - y)) f(y) dy + x(1 - e^{-\delta\tau(x)}) \int_{y=0}^{x} f(y) dy,$$

which is greater or equal to

$$\int_{y=\alpha}^{1} (r_1(x, y) - (1 - y)) f(y) dy + \int_{y=x}^{\alpha} (x - e^{-\delta\tau(y)}(1 - y)) f(y) dy,$$

which is equal to

$$\int_{y=x}^{1} (r_1(x, y) - (1 - y)) f(y) dy + \int_{y=x}^{\alpha} (x - e^{-\delta\tau(y)}(1 - y) - r_1(x, y) + (1 - y)) f(y) dy.$$

Notice that the first term is $g(x)$, which is strictly positive if $x > \theta$, and increasing with x. The second term, on the other hand, can be made as small as desired, by choosing x large enough, so as to be as closed as needed to α. Hence the expected net gain for such a type to reveal at zero is strictly positive, which contradicts the optimality of τ. This concludes the proof that $\tau(x) = 0$, for each $x > \theta$.

Finally, we prove that $\tau(\theta) = 0$. We have proved that $\theta = \alpha$. If $\tau(\theta) > 0$, then $\tau(x) \geq \tau(\theta)$, for all $x < \alpha$, by Step 2, and $\lim_{x \to \alpha_-} \tau(x) > 0$, which would contradict Step 3. Hence $\tau(\theta) = 0$, and we are done proving Step 4. □

Step 5. *τ is differentiable on $]0, \theta[$, and $\tau'(x) = \frac{(1-2x) f(x)}{\delta x F(x)}$, for all $x \in]0, \theta[$.*

Proof. Let $x \in]0, \theta[$. The expected net gain of revealing at $\tau(x+\epsilon)$ instead of $\tau(x)$ is equal to:

$$\int_{y=x}^{x+\epsilon} (xe^{-\delta\tau(x+\epsilon)} - (1-y)e^{-\delta\tau(y)}) f(y)dy + x(e^{-\delta\tau(x+\epsilon)} - e^{-\delta\tau(x)}) \int_{y=0}^{x} f(y)dy,$$

which is also equal to

$$-\int_{y=x}^{x+\epsilon} (1-y)e^{-\delta\tau(y)} f(y)dy + x(e^{-\delta\tau(x+\epsilon)} F(x+\epsilon) - e^{-\delta\tau(x)} F(x)).$$

In order for τ to be optimal, it must be that this expression is non-positive. Dividing by ϵ, and taking the limit when ϵ decreases to 0, we get:

$$-e^{-\delta\tau(x)}(1-2x)f(x) - x\delta \lim_{\epsilon\to 0_+} [\frac{\tau(x+\epsilon)-\tau(x)}{\epsilon}]e^{-\delta\tau(x)} F(x) \le 0.$$

A similar reasoning applied to the case that type $x+\epsilon$ is not better off by reporting at $\tau(x)$ gives

$$e^{-\delta\tau(x)}(1-2x)f(x) + x\delta \lim_{\epsilon\to 0_+} [\frac{\tau(x+\epsilon)-\tau(x)}{\epsilon}]e^{-\delta\tau(x)} F(x) \le 0.$$

Combining the two previous inequalities, we conclude that

$$\lim_{\epsilon\to 0_+} [\frac{\tau(x+\epsilon)-\tau(x)}{\epsilon}] = -\frac{(1-2x)f(x)}{\delta x F(x)}.$$

A similar reasoning with $\epsilon < 0$ implies that

$$\lim_{\epsilon\to 0_-} [\frac{\tau(x+\epsilon)-\tau(x)}{\epsilon}] = -\frac{(1-2x)f(x)}{\delta x F(x)},$$

which concludes the proof of this step. □

Step 6. $\tau = \tau^*$.

Proof. Step 4 establishes that $\tau = \tau^*$ on $[\theta, 1]$. Step 5 implies that $\tau = C + \tau^*$ on $[0, \theta[$, for some real number C. The fact that $\lim_{x\to\theta_-} \theta(x) = 0$, implies that $C = 0$, and establishes that $\tau = \tau^*$ on $[0, 1]$. □

Proof of Proposition 7. Given the characterization of the symmetric BNE in Proposition 6, we see that proving $\tau'(x) \le \tau(x)$, for each $x \in [0, 1]$, is equivalent to proving $\theta' \le \theta$, where θ and θ' are the thresholds defined in (4) for r and r' respectively. Suppose, to the contrary of what we want to prove, that $\theta' > \theta$. Then for any $\epsilon > 0$ small enough so that $\theta' - \epsilon > \theta$, we have $\int_{y=\theta'-\epsilon}^{1} (r_1(\theta'-\epsilon, y) - (1-y)) f(y)dy \ge 0$, by definition of θ. Since $r' \succeq r$, we must also have $\int_{y=\theta'-\epsilon}^{1} (r'_1(\theta'-\epsilon, y) - (1-y)) f(y)dy \ge 0$, but this contradicts the definition of θ'. Hence $\theta' \le \theta$, as desired. □

Proof of Proposition 8. Assume $\theta_S > \theta_D$ and let $\hat{\theta}$ be a type between θ_D and θ_S. Consider the static disclosure game first. Assume individual j uses the symmetric equilibrium strategy associated with the threshold θ_S. Then for type x of individual i, the expected net gain from disclosing, given by $xF(\theta_S) + \int_{\theta_S}^{1} [r_i(x, y) - (1-y)] f(y)dy$ is positive for all $x > \theta_S$ and negative for all $x < \theta_S$. In particular, it is negative for $x = \hat{\theta} < \theta_S$. Since $\hat{\theta} F(\theta_S)$ is strictly positive, it follows that $\int_{\theta_S}^{1} [r_i(\hat{\theta}, y) - (1-y)] f(y)dy < 0$. Because $\theta_S \le \frac{1}{2}$, we have that $1 - y > \hat{\theta}$ for all $\hat{\theta} \le y \le \theta_S$. Hence, $r_i(\hat{\theta}, y) \le (1-y)$ for all $\hat{\theta} \le y \le \theta_S$. Therefore, $\int_{\hat{\theta}}^{1} [r_i(\hat{\theta}, y) - (1-y)] f(y)dy < 0$. This contradicts the definition of $\theta_D < \hat{\theta}$ in Proposition 6. □

Proof of Proposition 9. The ex-ante expected sum of payoffs is equal to

$$1 - \theta_S^2 \tag{8}$$

since the sum of the individuals' payoffs equals 1 when at least one of them discloses his option, and 0 otherwise. Since both individuals are ex-ante symmetric, the ex-ante expected payoff of each is equal to $(1 - \theta_S^2)/2$.

A similar reasoning implies that the sum of individuals' ex-ante expected payoffs in the symmetric BNE of the dynamic game is equal to

$$1-\theta_D^2+\int\limits_{x=0}^{\theta_D}\int\limits_{y=0}^{\theta_D} e^{-\delta\tau(\max\{x,y\})}dxdy, \text{ where} \tag{9}$$

$$\tau(x)=\int\limits_{x}^{\theta_D}\frac{1-2y}{\delta y^2}dy=-\frac{1}{\delta\theta_D}+\frac{1}{\delta x}-\frac{2}{\delta}\ln\theta_D+\frac{2}{\delta}\ln x \tag{10}$$

for $x\leq\theta_D$ and uniform f.

Note that the last term in (9) is equal to

$$\int\limits_{x=0}^{\theta_D}\int\limits_{y=0}^{x} e^{-\delta\tau(x)}dxdy+\int\limits_{x=0}^{\theta_D}\int\limits_{y=x}^{\theta_D} e^{-\delta\tau(y)}dxdy. \tag{11}$$

Note that the second term in (11) may be rewritten as $\int_{y=0}^{\theta_D}\int_{x=0}^{y}e^{-\delta\tau(y)}dxdy$, and (11) itself may be rewritten as $2\int_{x=0}^{\theta_D}\int_{y=0}^{x}e^{-\delta\tau(x)}dxdy$ or $2\int_{x=0}^{\theta_D}xe^{-\delta\tau(x)}dx$. From (10) it follows that $e^{-\delta\tau(x)}=e^{1/\theta_D}\cdot e^{-1/x}\cdot\theta_D^2\cdot x^{-2}$. Therefore, (11) may also be rewritten as

$$2\int\limits_{x=0}^{\theta_D}[e^{1/\theta_D}\cdot\theta_D^2\cdot\frac{e^{-1/x}}{x}]dx=-2e^{1/\theta_D}\cdot\theta_D^2\cdot E_i(-\frac{1}{\theta_D}).$$

Substituting this expression into (9) tells that the sum of individuals' ex-ante expected payoffs in the symmetric BNE of the dynamic game is equal to

$$1-\theta_D^2-2e^{1/\theta_D}\cdot\theta_D^2\cdot E_i(-\frac{1}{\theta_D}). \tag{12}$$

Symmetry implies that the each individual's ex-ante expected payoff is 1/2 of this expression. Note it does not depend on the discount factor.

Substituting into (8) and (12) the equilibrium thresholds of the Nash solution are $(\sqrt{3}-1)/4$ and $1/4$ for the static and dynamic games, respectively (see the supplementary online appendix for details on this computation). Using these threshold values, one can check that the ex-ante expected payoffs for the Nash rule is superior in the static game.[20] □

Proof of Proposition 10. The proof methodology is comparable to that of Proposition 6, and is thus relegated to the supplementary online appendix. □

Proof of Proposition 11. (r^* IS REGULAR) r^* is clearly anonymous and efficient. Monotonicity follows at once from the definition of r^* in the following cases: (*i*) When starting from two points on the same side of the 45 degree line $u_2=u_1$ and changing only one of the points such that both still remain on the same side of $u_2=u_1$, and (*ii*) When starting from $(x,g(x))$ and $(z,g(z))$ with $g(x)>x$, $g(z)<z$ and $g(x)\geq z$, and changing $(z,f(z))$ into $(z',f(z'))$ while keeping $(x,f(x))$ fixed such that it is still the case that $g(x)\geq\max\{z',g(z')\}$.

By symmetry, Monotonicity must be established in one last case: starting from $(x,g(x))$ and $(z,g(z))$ with $g(x)>x$, $g(z)<z$, $g(x)>z$ and $g(z)>x$, and changing $(x,g(x))$ into $(x',g(x'))$ such that $g(x')>z$. To do this, we check the sign of the derivative of r_1^* with respect to its first component in this last region. It is helpful to do the following change of variable. For each $(x,g(x))$ falling in that last region, let α be the absolute value of the slope of the line joining $(z,g(z))$ to $(x,g(x))$. Vice versa, each $\alpha>1$ determines a unique $(x,g(x))$ that falls in that region (at the intersection of X and the line of slope $-\alpha$ that goes through $(z,g(z))$). Let $\delta=x+g(x)$ (note that this is the utilitarian surplus). Then, for each $\alpha>1$, we have $\delta(\alpha)/2=g(z)+\alpha(z-r_1^*(x(\alpha),g(z)))$, or $r_1^*(x(\alpha),g(z))=z-\frac{\delta(\alpha)-2g(z)}{2\alpha}$. Let now ϵ be any small strictly positive number. We have:

$$\frac{r_1^*(x(\alpha+\epsilon),g(z))-r_1^*(x(\alpha),g(z))}{\epsilon}=\frac{\delta(\alpha)\alpha+\delta(\alpha)\epsilon-2g(z)\epsilon-\alpha\delta(\alpha+\epsilon)}{2\alpha(\alpha+\epsilon)\epsilon}.$$

[20] Computing expected payoffs for the dynamic game requires knowing the value of the exponential integral at −4. Using numerical techniques, this value can be known with high precision. We can use for instance Harris' (1957) computation, which is precise up to the 18th decimal. Using comparable precision for the other operations needed to compute expected payoffs, we get 0.483253175473054852 in the static game, and 0.481646603118815964 in the dynamic game.

Taking the limit as epsilon tends to zero, this expression is equal to

$$-\frac{\delta'(\alpha)}{2\alpha}+\frac{\delta(\alpha)-2g(z)}{2\alpha^2}$$

(δ is differentiable because g is). Notice that $\delta(\alpha+\epsilon)$ is larger than the sum of the components of the vector at the intersection of this new line (going through $(z, g(z))$ and with angle $-\alpha-\epsilon$) and the vertical line going through $(x, g(x))$. This is so because the intersection of the new line with the utility frontier falls on the left of x, and the slope $\alpha+\epsilon$ is larger than 1 (i.e. any decrease in the first component is more than matched by an increase in the second component). The sum of the components of the vector associated to the new line is $x+g(x)+(z-x)\epsilon$. Therefore,

$$\delta'(\alpha)=\lim_{\epsilon\to 0}\frac{\delta(\alpha+\epsilon)-\delta(\alpha)}{\epsilon}\geq \lim_{\epsilon\to 0}\frac{x+g(x)+(z-x)\epsilon-x-g(x)}{\epsilon}=z-x.$$

Hence

$$\frac{dr_1^*(x(\alpha),g(z))}{d\alpha}\leq\frac{-\alpha(z-x)+\delta(\alpha)-2g(z)}{2\alpha^2}=\frac{x-g(z)}{2\alpha^2}\leq 0,$$

where the equality follows from the fact that $\alpha(z,x)=g(x)-g(z)$ and $\delta(\alpha)=x+g(x)$, and the last inequality follows from the fact that $x\leq g(z)$ (because $g(x)\geq z$, $g(z)<z$ and $g(x)>x$). Finally, $dx/d\alpha$ being strictly negative, it must be that r_1^* varies monotonically with x, as desired.

(r^* IS OPTIMAL) As pointed out at the beginning of Section 5.2, Proposition 4 is one of the results that carry over to any symmetric X with no Pareto comparisons. Hence Proposition 11 will follow after showing that $r^*\succeq r$, for all regular compromise rule r, which amounts to show $r_1^*(x,y)\geq r_1(x,y)$, for all $x\leq u^*$ and all $y\geq x$, where u^* is the real number such that $u^*=g(u^*)$. We may also assume without loss of generality that $y\leq g(x)$, as otherwise our argument applies by renaming $(x,g(x))$ $(g^{-1}(y),y)$, and vice-versa. We will be done after showing that monotonicity on r implies that $r_2(x,y)$ is no smaller than half the utilitarian surplus (since $r_1^*(x,y)$ is the first individual's largest feasible payoff under that constraint). The utilitarian surplus is achieved at $(x,g(x))$, since g is convex. Changing $(g^{-1}(y),y)$ into $(g^{-1}(x),x)$ does not increase the second individual's payoff (since $x\leq y$), while the second individual's payoff become equal to half the utilitarian surplus of the original problem (the new problem being solved by symmetry). □

Proof of Proposition 12. (*i*) By construction, the dual solution d is anonymous and ex-post efficient. To establish monotonicity, suppose we move from the payoff pair $(u,g(u))$ and $(g^{-1}(u'),u')$ to $(u^*,g(u^*))$ and $(g^{-1}(u'),u')$. If $u^*>u$, then $1-u^*<1-u$. Then $r_i(1-u^*,1-u')\leq r_i(1-u,1-u')$, because r is monotone. Hence, $d_i(u^*,u')\geq d_i(u,u')$. Essentially the same argument applies if we were to change $(g^{-1}(u'),u')$ holding fixed $(u,g(u))$.

(*ii*) Define ϕ as the value in $[0,1]$ that satisfies $\phi=g(\phi)$. We have to show that $d_1'(u,u')\geq d_1(u,u')$, for all $u\leq\min\{\phi,u'\}$. By definition of d, this is equivalent to showing that $r_1'(v,v')\leq r_1(v,v')$, where $v:=1-u$ and $v':=1-u'$. Since $r(v,v')$ and $r'(v,v')$ belong to the same segment with negative slope, this is equivalent to $r_2'(v,v')\geq r_2(v,v')$, which follows from the fact that $r'\succeq r$. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.geb.2015.02.016.

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