Online Appendix for "On Selecting the Right Agent"

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This Online Appendix contains proofs for the results discussed in Section 4.

S.1 Principal-renegotiation-proof equilibria

Proof of Proposition 9. Our first observation is that principal-renegotiation-proof PPEs have a very simple payoff structure.

Lemma S.1. *Consider a PPE where the principal gets the same (maximum) discounted expected payoff at the start of period, no matter the history. Then the principal gets the same expected equilibrium payoff within each stage game.*

Proof. Let X be the principal's discounted payoff at the start of any period. Let x be the principal's expected equilibrium payoff within that period. Then $X = (1-\delta)x + \delta X$, and hence $x = X$, which is independent of the history. □

This observation reduces the type of strategies that the principal employs in equilibrium. For example, it cannot be that there is a history after which the principal switches from not selecting the last agent who generated a low profit, to always selecting that agent, because these can give different expected payoffs to the principal within a stage game. Hence, characterizing the principal-renegotiation-proof PPEs reduces to finding which stage game behaviors lead to the same outcome and whether those stage games in the same 'equivalence class' can be sequenced in a way that forms

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a repeated-game equilibrium. It turns out that any PPE that satisfies our refinement gives the principal either his first-best payoff or his one-shot Nash payoff.

Note that each agent has four strategies in the stage game: propose regardless of qualification, don't propose regardless of qualification, propose only when qualified, and propose only when unqualified. There are thus sixteen combinations to consider for the agents. As for the principal, renegotiation-proofness implies that she gets the same discounted payoff at the beginning of any new round in the game, independently of what happened in the past. Hence it must be that she selects agents optimally in each repetition of the stage game taken individually (e.g. picking the discerning agent instead of the last resort in the MLR strategy profile when both agents make proposals). Otherwise, she would have a profitable unilateral deviation by picking the one that has a higher likelihood (given their equilibrium report strategies) of being qualified.

We already analyzed the following cases: (*i*) both agent propose regardless of their qualification, (*ii*) the most able agent proposes regardless of his qualification while the other agent does not propose regardless of his qualification, and (*iii*) one agent proposes only when qualified and the remaining agent proposes regardless of his qualification (note that these are two cases since the identity of the constant proposer can change). Cases (*i*) and (*ii*) correspond to the one-shot Nash equilibrium outcome, while case (*iii*) corresponds to the MLR strategy profile.

A fourth possible case is when every period both agents propose only when they are qualified. This generates the payoff $\big(1-(1-\theta_1)(1-\theta_2)\big)\big(\gamma H+(1-\gamma)L\big)$ to the principal, which is higher than the one-shot Nash payoff if $\min\{\theta_1,\theta_2\}>\big(\beta H+(1-\beta)L\big)/\big(\gamma H+(1-\gamma)L\big)$. We establish the following observation:

Lemma S.2. *Suppose there exists a PPE in which the agents propose if and only if they are qualified and the principal picks one of the proposing agents. Then the MLR strategy profile is also a PPE.*

Proof of Lemma S.2. We follow the same methodology as in the proof of Proposition 7. Using the same notation as in that proof, we let $\sigma_i(\emptyset)$ denote agent i's promised continuation payoff when no agent is selected.

Step 1. Deriving $\underline{\sigma}_1$. Suppose first $\underline{\sigma}_1$ is obtained when agent 1 is LR. to find $\underline{\sigma}_1$, minimize

$$(1-\theta_2)\theta_1\left[(1-\delta)+\delta\left(\gamma\sigma_1(1S)+(1-\gamma)\sigma_1(1F)\right)\right]+$$
$$(1-\theta_2)(1-\theta_1)\delta\sigma_1(\emptyset)+\theta_2\delta\left[\gamma\sigma_1(2S)+(1-\gamma)\sigma_1(2F)\right]$$

subject to the IC constraints that *both* agents do not propose when unqualified:[1]

$$\delta\left[\theta_1(\gamma\sigma_2(1S)+(1-\gamma)\sigma_2(1F))+(1-\theta_1)\sigma_2(\emptyset)\right]$$
$$\geq(1-\delta)+\delta\left(\beta\sigma_2(2S)+(1-\beta)\sigma_2(2F)\right),$$

for agent 2, and for agent 1: $\delta\sigma_1(\emptyset)\geq(1-\delta)+\delta\left[\beta\sigma_1(1S)+(1-\beta)\sigma_1(1F)\right]$. Since the sum of continuation payoffs is always $1-(1-\theta_1)(1-\theta_2)$, we can rewrite agent 2's IC as

$$\delta\beta\sigma_1(2S)+\delta(1-\beta)\sigma_1(2F)\geq(1-\delta)+\delta\theta_1\gamma\sigma_1(1S)+\delta(1-\gamma)\theta_1\sigma_1(1F)+\delta(1-\theta_1)\sigma_1(\emptyset).$$

Hence, we can decrease $\sigma_1(1S),\sigma_1(1F)$ all the way to $\underline{\sigma}_1$ (reduces the continuation payoff and can only relax the IC). We then have the following problem:

$$\min\ (1-\theta_2)\theta_1\left[(1-\delta)+\delta\underline{\sigma}_1\right]+(1-\theta_2)(1-\theta_1)\delta\sigma_1(\emptyset)+\theta_2\delta\left[\gamma\sigma_1(2S)+(1-\gamma)\sigma_1(2F)\right]$$

subject to the feasibility constraint that continuation payoffs lie in $[\underline{\sigma}_1,\overline{\sigma}_1]$, and the IC's

$$\delta\left(\beta\sigma_1(2S)+(1-\beta)\sigma_1(2F)\right)\quad\geq\quad(1-\delta)+\delta\theta_1\underline{\sigma}_1+\delta(1-\theta_1)\sigma_1(\emptyset)$$

and $\delta\sigma_1(\emptyset)\geq(1-\delta)+\delta\underline{\sigma}_1$. Substituting the latter IC (which must clearly bind) into the former, and also into the objective function, we wish to minimize

$$(1-\theta_2)\left[(1-\delta)+\delta\underline{\sigma}_1\right]+\theta_2\delta\left[\gamma\sigma_1(2S)+(1-\gamma)\sigma_1(2F)\right]$$

subject to feasibility and $\sigma_1(2F)=\frac{1-\delta}{\delta(1-\beta)}(2-\theta_1)+\frac{\underline{\sigma}_1}{1-\beta}-\frac{\beta}{1-\beta}\sigma_1(2S)$. Plugging this back into the objective function we obtain that the coefficient on $\sigma_1(2S)$ is $\frac{\gamma-\beta}{1-\beta}>0$. We therefore wish to reduce $\sigma_1(2S)$ as much as possible, noting that a decrease in

[1] As in the symmetric case, we ignore the remaining constraints, which will turn out to be without loss.

$\sigma_1(2S)$ yields an increase in $\sigma_1(2F)$. There are therefore two cases to consider:

Case 1. $\sigma_1(2S) = \underline{\sigma}_1$ *and* $\sigma_1(2F) = \frac{1-\delta}{\delta(1-\beta)}(2-\theta_1) + \underline{\sigma}_1 \leq \overline{\sigma}_1$. In this case it must hold that $\overline{\sigma}_1 - \underline{\sigma}_1 \geq \frac{1-\delta}{\delta(1-\beta)}(2-\theta_1)$. Setting $\underline{\sigma}_1$ equal to the objective in the minimization problem, we obtain $\underline{\sigma}_1 = (1-\theta_2) + \theta_2 \frac{1-\gamma}{1-\beta}(2-\theta_1)$. The necessary condition for Case 1 is therefore:

$$(1-\theta_2) + \theta_2 \frac{1-\gamma}{1-\beta}(2-\theta_1) + \frac{1-\delta}{\delta(1-\beta)}(2-\theta_1) \leq \overline{\sigma}_1.$$

To check when it is satisfied, we will consider later below the problem of maximizing 1's continuation payoff.

Case 2. $\sigma_1(2F) = \overline{\sigma}_1$ *and* $\sigma_1(2S) = \frac{1-\delta}{\delta\beta}(2-\theta_1) + \frac{\underline{\sigma}_1}{\beta} - \frac{1-\beta}{\beta}\overline{\sigma}_1 \in [\underline{\sigma}_1, \overline{\sigma}_1]$. Setting $\underline{\sigma}_1$ equal to the objective in the minimization problem, we obtain that

$$\underline{\sigma}_1 = \frac{(1-\delta)\left[(1-\theta_2) + \theta_2 \frac{\gamma}{\beta}(2-\theta_1)\right] - \delta\theta_2\overline{\sigma}_1 \left[\frac{\gamma-\beta}{\beta}\right]}{1-\delta\left[(1-\theta_2) + \theta_2\frac{\gamma}{\beta}\right]}. \tag{S.1}$$

Step 2. Deriving $\overline{\sigma}_1$. We now maximize 1's continuation payoff. Suppose first that this occurs when 1 is discerning. We maximize

$$\begin{aligned} &\theta_1\left[(1-\delta) + \delta\left(\gamma\sigma_1(1S) + (1-\gamma)\sigma_1(1F)\right)\right] \\ &+ (1-\theta_1)\delta\left[\theta_2(\gamma\sigma_1(2S) + (1-\gamma)\sigma_1(2F)) + (1-\theta_2)\sigma_1(\emptyset)\right] \end{aligned}$$

subject to the IC that neither agent wants to propose when unqualified:

$$\begin{aligned} &\delta\left[\theta_2(\gamma\sigma_1(2S) + (1-\gamma)\sigma_1(2F)) + (1-\theta_2)\sigma_1(\emptyset)\right] \geq (1-\delta) + \delta\left(\beta\sigma_1(1S) + (1-\beta)\sigma_1(1F)\right) \\ &\delta\sigma_2(\emptyset) \geq (1-\delta) + \delta\left(\beta\sigma_2(2S) + (1-\beta)\sigma_2(2F)\right) \end{aligned}$$

and $\sigma_1 \in [\underline{\sigma}_1, \overline{\sigma}_1]$. Using the fact that continuation payoffs following each event must sum to $1-(1-\theta_1)(1-\theta_2)$, we rewrite agent 2's IC as $\delta\left(\beta\sigma_1(2S) + (1-\beta)\sigma_1(2F)\right) \geq (1-\delta) + \delta\sigma_1(\emptyset)$. Setting $\sigma_1(2S), \sigma_1(2F) = \overline{\sigma}_1$ (increases objective and only relaxes IC), we wish to maximize

$$\theta_1\left[(1-\delta) + \delta\left(\gamma\sigma_1(1S) + (1-\gamma)\sigma_1(1F)\right)\right] + (1-\theta_1)\theta_2\delta\overline{\sigma}_1 + (1-\theta_1)(1-\theta_2)\delta\sigma_1(\emptyset)$$

subject to feasibility,

$$\delta\theta_2\overline{\sigma}_1 + (1-\theta_2)\delta\sigma_1(\emptyset) \quad \geq \quad (1-\delta) + \delta\left(\beta\sigma_1(1S) + (1-\beta)\sigma_1(1F)\right),$$

and $\delta\overline{\sigma}_1 \geq (1-\delta) + \delta\sigma_1(\emptyset)$. Since the latter must bind, plugging into the other the first IC, we obtain $\delta\overline{\sigma}_1 \geq (1-\delta)(2-\theta_2) + \delta\left(\beta\sigma_1(1S) + (1-\beta)\sigma_1(1F)\right)$, which clearly must bind. Therefore the objective of maximization becomes:

$$(1-\delta)\left(\theta_1 - (1-\theta_1)(1-\theta_2)\right) + \theta_1\delta\left(\gamma\sigma_1(1S) + (1-\gamma)\sigma_1(1F)\right) + (1-\theta_1)\delta\overline{\sigma}_1.$$

To solve the maximization problem, we must increase $\sigma_1(1S)$ as much as possible (intuitively, increase agent 1's payoff when he is discerning and succeeds), and run into two cases:

Case 3. $\sigma_1(1S) = \overline{\sigma}_1$ *and* $\sigma_1(1F) = \overline{\sigma}_1 - \frac{(1-\delta)}{\delta(1-\beta)}(2-\theta_2) \geq \underline{\sigma}_1$. Setting $\overline{\sigma}_1$ equal to the objective in the maximization, $\overline{\sigma}_1 = \theta_1(2-\theta_2)\frac{\gamma-\beta}{1-\beta} - (1-\theta_2)$. So the necessary condition is

$$\theta_1(2-\theta_2)\frac{\gamma-\beta}{1-\beta} - (1-\theta_2) - \frac{(1-\delta)}{\delta(1-\beta)}(2-\theta_2) \geq \underline{\sigma}_1.$$

Case 4. $\sigma_1(1F) = \underline{\sigma}_1$ *and* $\sigma_1(1S) = \frac{\overline{\sigma}_1}{\beta} - \frac{(1-\beta)\underline{\sigma}_1}{\beta} - \frac{1-\delta}{\delta\beta}(2-\theta_2) \in [\underline{\sigma}_1, \overline{\sigma}_1]$. Plugging into the objective yields,

$$\overline{\sigma}_1 = \frac{(1-\delta)\left(\theta_1 - (1-\theta_1)(1-\theta_2) - \theta_1(2-\theta_2)\frac{\gamma}{\beta}\right) - \delta\underline{\sigma}_1\theta_1\left(\frac{\gamma}{\beta} - 1\right)}{1 - \delta\left(\theta_1\frac{\gamma}{\beta} + (1-\theta_1)\right)}. \tag{S.2}$$

Since $\theta_1 - (1-\theta_1)(1-\theta_2) - \theta_1(2-\theta_2)\frac{\gamma}{\beta} < 0$, it must be that $1 - \delta\left(\theta_1\frac{\gamma}{\beta} + (1-\theta_1)\right) < 0$.

Step 3. Combining cases 1 and 3. From Case 3, $\overline{\sigma}_1 = (\theta_1 - (1-\theta_1)(1-\theta_2)) - \theta_1(2-\theta_2)\frac{1-\gamma}{1-\beta}$ and $\overline{\sigma}_1 - \underline{\sigma}_1 \geq \frac{1-\delta}{\delta(1-\beta)}(2-\theta_2)$, and from Case 1 we have $\underline{\sigma}_1 = (1-\theta_2) + \theta_2\frac{1-\gamma}{1-\beta}(2-\theta_1)$ and $\overline{\sigma}_1 - \underline{\sigma}_1 \geq \frac{1-\delta}{\delta(1-\beta)}(2-\theta_1)$. Therefore, we have

$$\overline{\sigma}_1 - \underline{\sigma}_1 = 2\left(\theta_1 + \theta_2 - \theta_1\theta_2\right)\left(\frac{\gamma-\beta}{1-\beta}\right) - 2 + \theta_1\theta_2.$$

And the combined necessary condition for the two cases is

$$\delta \;\; \geq \;\; \max_{i\in\{1,2\}} \left\{ \frac{1}{\frac{g(\theta_1,\theta_2,\gamma,\beta)}{2-\theta_i}+1} \right\}, \tag{S.3}$$

where $g(\theta_1,\theta_2,\gamma,\beta) \; = \; 2\left(\theta_1+\theta_2-\theta_1\theta_2\right)(\gamma-\beta)-(1-\beta)(2-\theta_1\theta_2)$. Note that the effective constraint is the one with the smaller θ_1.

Assume first that $\theta_1 \leq \theta_2$. We want to verify that the necessary constraint for the candidate equilibrium (proposing only when qualified) is more restrictive than the one for MLR, i.e., that

$$\frac{2-\theta_1}{g(\theta_1,\theta_2,\gamma,\beta)+2-\theta_1} > \frac{1}{\beta+(\theta_1+\theta_2)(\gamma-\beta)}.$$

This inequality holds if and only if $(\theta_2-\theta_1)(\gamma-\beta) > -(1-\beta)(1-\theta_2)$, which holds since $\gamma > \beta$. The analogous argument holds for $\theta_1 > \theta_2$. It follows that indeed the necessary conditions of cases 1 and 3 are more stringent than the condition that assures that MLR is a PPE.

Step 4. Combining cases 2 and 4. From Case 2 we have (S.1) and from Case 4 we have (S.2). Combining the two yields:

$$\overline{\sigma}_1 - \underline{\sigma}_1 = \frac{(1-\delta)\left[2\left(\theta_1+\theta_2-\theta_1\theta_2\right)\left(\frac{\gamma}{\beta}-1\right)+2-\theta_1\theta_2\right]}{\delta(\theta_1+\theta_2)\left(\frac{\gamma}{\beta}-1\right)-(1-\delta)},$$

and the necessary conditions for these two cases reduce to

$$\max_{i\in\{1,2\}}\left\{\frac{(1-\delta)(2-\theta_i)}{\delta}\right\} \leq \overline{\sigma}_1 - \overline{\sigma}_1 \leq \min_{i\in\{1,2\}}\left\{\frac{(1-\delta)(2-\theta_i)}{\delta(1-\beta)}\right\}.$$

Suppose first that $\theta_2 \geq \theta_1$. Then it suffices to check the upper bound $\frac{(1-\delta)(2-\theta_2)}{\delta(1-\beta)}$ and the lower bound $\frac{(1-\delta)(2-\theta_1)}{\delta}$. Starting with the upper bound:

$$\overline{\sigma}_1 - \underline{\sigma}_1 = \frac{(1-\delta)\left[2\left(\theta_1+\theta_2-\theta_1\theta_2\right)\left(\frac{\gamma}{\beta}-1\right)+2-\theta_1\theta_2\right]}{\delta(\theta_1+\theta_2)\left(\frac{\gamma}{\beta}-1\right)-(1-\delta)} \leq \frac{(1-\delta)(2-\theta_2)}{\delta(1-\beta)}$$

which can be rewritten as

$$\delta \geq \frac{2-\theta_2}{\left(\frac{\gamma}{\beta}-1\right)\left[\left(2-\theta_2\right)\left(\theta_1+\theta_2\right)-(1-\beta)2\left(\theta_1+\theta_2-\theta_1\theta_2\right)\right]+\left(2-\theta_2\right)-(1-\beta)\left(2-\theta_1\theta_2\right)}.$$

We want to show that that this constraint is more restrictive than the one for MLR. That is, that the RHS of the last inequality is greater than $\frac{1}{\beta+(\theta_1+\theta_2)(\gamma-\beta)}$. After some algebra, it can be shown that this is equivalent to $(\theta_2-\theta_1)\frac{\gamma}{\beta}(\beta-1)<(1-\theta_2)(1-\beta)$, which is clearly satisfied since the LHS is negative. So for $\theta_2 \geq \theta_1$ it must be that the combination of cases 2 and 4 hold only under conditions more restrictive than the equilibrium condition for MLR; equivalently, the condition for the existence of a first-best equilibrium (there is no need to check the lower bound).

Next suppose $\theta_2<\theta_1$. Then it suffices to check the upper bound $\frac{(1-\delta)(2-\theta_1)}{\delta(1-\beta)}$ and the lower bound $\frac{(1-\delta)(2-\theta_2)}{\delta}$. As before, we start with the upper bound:

$$\overline{\sigma}_1-\underline{\sigma}_1=\frac{(1-\delta)\left[2\left(\theta_1+\theta_2-\theta_1\theta_2\right)\left(\frac{\gamma}{\beta}-1\right)+2-\theta_1\theta_2\right]}{\delta(\theta_1+\theta_2)\left(\frac{\gamma}{\beta}-1\right)-(1-\delta)} \leq \frac{(1-\delta)(2-\theta_1)}{\delta(1-\beta)},$$

or equivalently

$$\delta \geq \frac{2-\theta_1}{(2-\theta_1)(\theta_1+\theta_2)\left(\frac{\gamma}{\beta}-1\right)+(2-\theta_1)-(1-\beta)\left[2\left(\theta_1+\theta_2-\theta_1\theta_2\right)\left(\frac{\gamma}{\beta}-1\right)+2-\theta_1\theta_2\right]}. \tag{S.4}$$

We therefore want to show that the RHS of this last inequality is greater than $\frac{1}{\beta+(\theta_1+\theta_2)(\gamma-\beta)}$. This is equivalent to the inequality $(1-\beta)(1-\theta_1)>\frac{\gamma}{\beta}\left(\theta_1-\theta_2\right)(\beta-1)$, which holds since the RHS is negative. It follows that the conditions for cases 2 and 4 are more stringent than the condition for attaining the first-best in PPE.

Step 5. Combining cases 1 and 4. From Case 1 we have $\underline{\sigma}_1=(1-\theta_2)+\theta_2\frac{1-\gamma}{1-\beta}(2-\theta_1)$, and $\overline{\sigma}_1-\underline{\sigma}_1 \geq \frac{1-\delta}{\delta(1-\beta)}(2-\theta_1)$, and from Case 4 we have

$$\overline{\sigma}_1=\frac{(1-\delta)\left((1-\theta_1)(1-\theta_2)-\theta_1+\theta_1(2-\theta_2)\frac{\gamma}{\beta}\right)+\delta\underline{\sigma}_1\theta_1\left(\frac{\gamma}{\beta}-1\right)}{\delta\left(\theta_1\frac{\gamma}{\beta}+(1-\theta_1)\right)-1} \tag{S.5}$$

and $\overline{\sigma}_1-\underline{\sigma}_1 \in \left[\frac{(1-\delta)(2-\theta_2)}{\delta}, \frac{(1-\delta)(2-\theta_2)}{\delta(1-\beta)}\right]$. Combining these, we get the necessary condition

$$\frac{(2-\theta_1)[1-\theta_2(\frac{\gamma-\beta}{1-\beta})]-\theta_1+\theta_1(2-\theta_2)\frac{\gamma}{\beta}}{\delta\left(\theta_1\frac{\gamma}{\beta}+(1-\theta_1)\right)-1}\leq\frac{(2-\theta_2)}{\delta(1-\beta)}.$$

Note that an implicit requirement for Case 4 is that $\delta\left(\theta_1\frac{\gamma}{\beta}+(1-\theta_1)\right)-1>0$, since the numerator in the expression (S.5) is positive and hence the denominator must also be positive to guarantee $\overline{\sigma}_1>0$. Therefore, rearranging the necessary condition above yields:

$$\delta\geq\frac{2-\theta_2}{\theta_1(2-\theta_2)\gamma+(1-\theta_1)(2-\theta_2)+\theta_2(2-\theta_1)(\gamma-\beta)-2(1-\beta)(1-\theta_1)}. \tag{S.6}$$

We want to show that the RHS of (S.6) is greater than $\frac{1}{\beta+(\theta_1+\theta_2)(\gamma-\beta)}$. The last inequality, after some algebra, is equivalent to $(1-\beta)-\theta_1(1-\gamma)-(\gamma-\beta)\theta_2>0$, which clearly holds.

Step 6. Combining cases 2 and 3. From Case 3 we have

$$\overline{\sigma}_1=(\theta_1-(1-\theta_1)(1-\theta_2))-\theta_1(2-\theta_2)\frac{1-\gamma}{1-\beta}$$

and $\overline{\sigma}_1-\underline{\sigma}_1\geq\frac{1-\delta}{\delta(1-\beta)}(2-\theta_2)$, and from Case 2 we have (S.1) and $\overline{\sigma}_1-\underline{\sigma}_1\in\left[\frac{(1-\delta)(2-\theta_1)}{\delta},\frac{(1-\delta)(2-\theta_1)}{\delta(1-\beta)}\right]$. Solving for $\overline{\sigma}_1-\underline{\sigma}_1$, we get

$$\overline{\sigma}_1-\underline{\sigma}_1=(1-\delta)\frac{2(1-\theta_2)+\frac{\gamma}{\beta}\theta_2(2-\theta_1)-\theta_1(2-\theta_2)\frac{\gamma-\beta}{1-\beta}}{\delta\left[(1-\theta_2)+\theta_2\frac{\gamma}{\beta}\right]-1}.$$

One necessary condition is therefore:

$$\frac{2(1-\theta_2)+\frac{\gamma}{\beta}\theta_2(2-\theta_1)-\theta_1(2-\theta_2)\frac{\gamma-\beta}{1-\beta}}{\delta\left[(1-\theta_2)+\theta_2\frac{\gamma}{\beta}\right]-1}\leq\frac{(2-\theta_1)}{\delta(1-\beta)}.$$

Suppose first that $\delta\left[(1-\theta_2)+\theta_2\frac{\gamma}{\beta}\right]-1>0$, i.e., $\delta>\frac{1}{(1-\theta_2)+\theta_2\frac{\gamma}{\beta}}$. Then after some algebra we obtain that the necessary condition can be rewritten as

$$2-\theta_1 \leq \delta\Big((2-\theta_1)(1-\theta_2)+(2-\theta_1)\theta_2\frac{\gamma}{\beta}+\theta_1(2-\theta_2)(\gamma-\beta)-(1-\beta)2(1-\theta_2) - \frac{\gamma}{\beta}\theta_2(2-\theta_1)+\gamma\theta_2(2-\theta_1)\Big).$$

Note that if the RHS is negative, we are done, since a necessary condition for cases 2 and 3 cannot be satisfied. We can therefore divide to obtain

$$\delta \geq \frac{2-\theta_1}{(2-\theta_1)(1-\theta_2)+\theta_1(2-\theta_2)(\gamma-\beta)-(1-\beta)2(1-\theta_2)+\gamma\theta_2(2-\theta_1)}. \tag{S.7}$$

We want to show that the RHS of (S.7) is greater than $\frac{1}{\beta+(\theta_1+\theta_2)(\gamma-\beta)}$. This inequality reduces to $(1-\beta)-(\gamma-\beta)\theta_1-\theta_2(1-\gamma)>0$, which holds since $(1-\beta)-(\gamma-\beta)\theta_1-\theta_2(1-\gamma)>(1-\beta)-(\gamma-\beta)-(1-\gamma)=0$. It remains to consider the case $\delta\left[(1-\theta_2)+\theta_2\frac{\gamma}{\beta}\right]-1<0$, i.e., $\delta<\frac{1}{(1-\theta_2)+\theta_2\frac{\gamma}{\beta}}$. Recall that another necessary condition for cases 2 and 3 is that

$$\overline{\sigma}_1-\underline{\sigma}_1=\frac{2(1-\theta_2)+\frac{\gamma}{\beta}\theta_2(2-\theta_1)-\theta_1(2-\theta_2)\frac{\gamma-\beta}{1-\beta}}{\delta\left[(1-\theta_2)+\theta_2\frac{\gamma}{\beta}\right]-1} \geq \frac{(2-\theta_1)}{\delta}.$$

Rearranging, since $\delta\left[(1-\theta_2)+\theta_2\frac{\gamma}{\beta}\right]-1<0$, we get

$$2-\theta_1 \leq \delta\theta_1\left((2-\theta_2)\frac{\gamma-\beta}{1-\beta}-(1-\theta_2)\right).$$

If $(2-\theta_2)\frac{\gamma-\beta}{1-\beta}-(1-\theta_2)<0$, we are done. Assuming $(2-\theta_2)\frac{\gamma-\beta}{1-\beta}-(1-\theta_2)>0$, we get

$$\frac{2-\theta_1}{\theta_1\left[(2-\theta_2)\frac{\gamma-\beta}{1-\beta}-(1-\theta_2)\right]} \leq \delta. \tag{S.8}$$

We therefore get a contradiction if we show that

$$\frac{2-\theta_1}{\theta_1\left[(2-\theta_2)\frac{\gamma-\beta}{1-\beta}-(1-\theta_2)\right]} > \frac{1}{(1-\theta_2)+\theta_2\frac{\gamma}{\beta}} \tag{S.9}$$

since this, together with (S.8), contradicts $\delta < \frac{1}{(1-\theta_2)+\theta_2\frac{\gamma}{\beta}}$. Indeed, (S.9) simplifies to

$$(2-\theta_2)\left(1-\theta_1\frac{\gamma-\beta}{1-\beta}\right)+\theta_2\left((2-\theta_1)\frac{\gamma}{\beta}-1\right)>0,$$

which holds since $1-\theta_1\frac{\gamma-\beta}{1-\beta}>0$ and $(2-\theta_1)\frac{\gamma}{\beta}-1>0$.

Step 7. Verifying the postulated configuration of roles.

Claim 1. *$\underline{\sigma}_1$ is attained when agent 1 is last-resort.*

Proof. Assume, by contradiction, that $\underline{\sigma}_1$ is attained when agent 1 is discerning,

$$\begin{aligned}\underline{\sigma}_1 \geq \min \theta_1[(1-\delta)+\delta\left(\gamma\sigma_1^D(1S)+(1-\gamma)\sigma_1^D(1F)\right)\\ +(1-\theta_1)\delta\left(\theta_2\left(\gamma\sigma_1^D(2S)+(1-\gamma)\sigma_1^D(2F)\right)+(1-\theta_2)\sigma_1^D(\emptyset)\right).\end{aligned}$$

The IC constraint of agent 1 for not proposing when unqualified is:

$$\begin{aligned}&\delta\left(\theta_2\left(\gamma\sigma_1^D(2S)+(1-\gamma)\sigma_1^D(2F)\right)+(1-\theta_2)\sigma_1^D(\emptyset)\right)\\ &\geq (1-\delta)+\delta\left(\beta\sigma_1^D(1S)+(1-\beta)\sigma_1^D(1F)\right).\end{aligned}$$

Hence, we must have:

$$\begin{aligned}&\theta_1\left((1-\delta)+\delta\left(\gamma\sigma_1^D(1S)+(1-\gamma)\sigma_1^D(1F)\right)\right)\\ &+(1-\theta_1)\delta\left(\theta_2\left(\gamma\sigma_1^D(2S)+(1-\gamma)\sigma_1^D(2F)\right)+(1-\theta_2)\sigma_1^D(\emptyset)\right)\\ &\geq \theta_1\left((1-\delta)+\delta\left(\gamma\sigma_1^D(1S)+(1-\gamma)\sigma_1^D(1F)\right)\right)\\ &+(1-\theta_1)\left((1-\delta)+\delta\left(\beta\sigma_1^D(1S)+(1-\beta)\sigma_1^D(1F)\right)\right)\geq(1-\delta)+\delta\underline{\sigma}_1.\end{aligned}$$

But this implies that $\underline{\sigma}_1 \geq (1-\delta)+\delta\underline{\sigma}_1$ or that $\underline{\sigma}_1 \geq 1$, a contradiction.

Claim 2. *$\overline{\sigma}_1$ is attained when agent 1 is discerning.*

Proof. Assume, by contradiction, that $\overline{\sigma}_1$ is attained when agent 1 is last-resort. Then

$$\begin{aligned}\overline{\sigma}_1 \leq \max(1-\theta_2)\left(\theta_1\left((1-\delta)+\delta\left(\gamma\sigma_1^{LR}(1S)+(1-\gamma)\sigma_1^{LR}(1F)\right)\right)+\delta(1-\theta_1)\sigma_1^{LR}(\emptyset)\right)\\ +\delta\theta_2\left(\gamma\sigma_1^{LR}(2S)+(1-\gamma)\sigma_1^{LR}(2F)\right)\end{aligned}$$

The IC constraint of the discerning agent 2 for not proposing when unqualified is:

$$(1-\delta)+\delta\left(\beta\sigma_2^D(2S)+(1-\beta)\sigma_2^D(2F)\right)\le\delta\theta_1\left(\gamma\sigma_2^D(1S)+(1-\gamma)\sigma_2^D(1F)\right)+\delta(1-\theta_1)\sigma_2^D(\emptyset).$$

Since $\sigma_1^{LR}(x)+\sigma_2^D(x)=1$ for $x\in\{1S,1F,2S,2F\}$, we can write this constraint as:

$$(1-\delta)+\delta\theta_1\left(\gamma\sigma_1^{LR}(1S)+(1-\gamma)\sigma_1^{LR}(1F)\right)+\delta(1-\theta_1)\sigma_1^{LR}(\emptyset)\le\delta\left(\beta\sigma_1^{LR}(2S)+(1-\beta)\sigma_1^{LR}(2F)\right).$$

Therefore:

$$\begin{aligned}
&(1-\theta_2)\left(\theta_1\left((1-\delta)+\gamma\delta\sigma_1^{LR}(1S)+(1-\gamma)\delta\sigma_1^{LR}(1F)\right)+(1-\theta_1)\delta\sigma_1^{LR}(\emptyset)\right)\\
&\quad+\theta_2\delta\left(\gamma\sigma_1^{LR}(2S)+(1-\gamma)\sigma_1^{LR}(2F)\right)\\
&\le-(1-\theta_2)(1-\theta_1)(1-\delta)+(1-\theta_2)\delta\left(\beta\sigma_1^{LR}(2S)+(1-\beta)\sigma_1^{LR}(2F)\right)\\
&\quad+\theta_2\delta\left(\gamma\sigma_1^{LR}(2S)+(1-\gamma)\sigma_1^{LR}(2F)\right)\\
&\le\delta\overline{\sigma}_1.
\end{aligned}$$

But this implies that $\overline{\sigma}_1\le\delta\overline{\sigma}_1$ or $1\le\delta$. □

This implies that if the principal's first-best cannot be attained in a PPE, then there cannot be a PPE where the agents propose if and only if they are qualified. It is straightforward to verify that none of the remaining cases lead to an expected stage-game payoff for the principal that is higher than that of the one-shot Nash. ■

S.2 Non-existence of PPE where only qualified agents propose

Proof of Proposition 8. Recall the necessary conditions derived for the four cases in the proof of Lemma S.2 (inequalities (S.3)-(S.4)). When $\theta_1=\theta_2=\theta$ the lower bound for cases (1+3), (2+3) and (1+4) are exactly the same, and lower than that of (2+4). Hence, a necessary condition for this case is:

$$\delta\ge\frac{1}{2\theta(\gamma-\beta)-(1-\beta)(\frac{2-\theta^2}{2-\theta})+1}.$$

This requires $1-\frac{1-\gamma}{1-\beta} > \frac{2-\theta^2}{2\theta(2-\theta)}$. But since $\frac{2-\theta^2}{2\theta(2-\theta)} > 1$, this condition can never hold. ■

S.3 General profit distributions

Recall the definitions of Q, U, $\underline{y}$, $\bar{y}$, γ^*, β^*, and the adjusted-MLR strategy from the main text. We now examine how the punishment set Y should be chosen to sustain the equilibrium, when possible. Consider, for instance, the model with uncertain abilities in $[\underline{\theta}, 1]^2$ that we characterized in Proposition 2. As seen from that result, the first-best is achievable in a belief-free equilibrium if and only if for all agents i,

$$\delta \geq \frac{1}{\beta^* + 2\underline{\theta}(\gamma^* - \beta^*)} = \frac{1}{1 - P_U(Y) + 2\underline{\theta}(P_U(Y) - P_Q(Y))}. \tag{S.10}$$

First, it is clear from (S.10) that the punishment set Y must be more likely for an unqualified agent than a qualified one (i.e., $\gamma^* > \beta^*$). Intuitively, incentive conditions would be impossible to satisfy if this were not the case. Moreover, the punishment set must be chosen so that the denominator in (S.10) is strictly larger than one, or equivalently:

$$2\underline{\theta} > \frac{P_U(Y)}{P_U(Y) - P_Q(Y)} = \frac{P_U(Y)/P_Q(Y)}{P_U(Y)/P_Q(Y) - 1}. \tag{S.11}$$

The smallest $\underline{\theta}$ for which this is possible is obtained by picking Y to maximize the likelihood ratio $P_U(Y)/P_Q(Y)$ that the punishment set comes from an unqualified agent versus a qualified one. If there exists a profit level y in the support of B but not of G, then this ratio is made arbitrarily large by setting $Y = [y-\varepsilon, y+\varepsilon]$ for small enough ε.

What happens when unqualified agents cannot be identified with certainty (i.e., the support of U is contained in the support of Q)? Suppose, for instance, that U and Q have continuous densities u and q satisfying the monotone likelihood ratio property, with $u(y)/q(y)$ decreasing in y.[2] Assuming $\underline{y}$ is in the support of u, the maximum of $P_U(Y)/P_Q(Y)$ can be shown to be $\lim_{y\to\underline{y}} u(y)/q(y)$.[3] Hence a belief-free equilibrium achieves the first-best if $2\underline{\theta} > \frac{\lim_{y\to\underline{y}} u(y)/q(y)}{\lim_{y\to\underline{y}} u(y)/q(y)-1}$. If the likelihood ratio goes to infinity

[2] The case of probability mass functions u, q satisfying the monotone likelihood ratio property is similar.

[3] For $Y = [\underline{y}, y]$, we have $\lim_{y\to\underline{y}} P_U(Y)/P_Q(Y) = \lim_{y\to\underline{y}} U(y)/Q(y) = \lim_{y\to\underline{y}} u(y)/Q(y)$ by l'Hôpital's rule. Moreover, for any other Y with positive measure under U (and thus Q, by the

as y decreases to $\underline{y}$, then for any $\underline{\theta} > 1/2$, one can find a y^* low enough to guarantee that the first-best can be achieved in a belief-free way with $Y = [\underline{y}, y^*]$ for sufficiently patient agents.

We may want to select the punishment set so that first-best is achievable for the largest range of discount factors. In view of (S.10), we would choose Y to maximize the objective:

$$-P_U(Y) + 2\underline{\theta}[P_U(Y) - P_Q(Y)] = \int_{y \in Y} \Big((2\underline{\theta} - 1)u(y) - 2\underline{\theta}q(y)\Big)dy.$$

To that end, a profit level y should be included in the punishment set if and only if

$$\frac{u(y)}{q(y)} \geq \frac{2\underline{\theta}}{2\underline{\theta} - 1}. \tag{S.12}$$

Under the monotone likelihood ratio property, the optimal punishment set will be an interval $Y = [\underline{y}, y^*]$, where y^* satisfies condition (S.12) with equality.

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inclusion of the support),

$$\frac{P_U(Y)}{P_Q(Y)} = \frac{\int_{y \in Y} u(y)dy}{\int_{y \in Y} q(y)dy} = \frac{\int_{y \in Y} \frac{u(y)}{q(y)} q(y)dy}{\int_{y \in Y} q(y)dy} \leq \lim_{y \to \underline{y}} \frac{u(y)}{q(y)}.$$